Non-equilibrium Transport and Relaxation in Diffusive Nanowires with Kondo Impurities

Johann Kroha[1], Achim Rosch[2], Jens Paaske[2,3], and Peter Wölfle[2]

[1] Physikalisches Institut, Universität Bonn, Nussallee 12, 53115 Bonn, Germany
[2] Institut für Theorie der Kondensierten Materie, Universität Karlsruhe, Postfach 6980, 76128 Karlsruhe, Germany
[3] Ørsted Laboratory, Niels Bohr Institute, University of Copenhagen, Universitetsparken 5, DK-2100 Copenhagen, Denmark

Abstract. Combining non-equilibrium transport with spectroscopic measurements provides a unique tool for the investigation of the microscopic processes in mesoscopic conductors. Experiments on resistive quantum wires show that the non-equilibrium quasiparticle distribution function $f(E, V)$ as a function of the quasiparticle energy E approximately obeys the scaling property, $f(E, V) = f(E/V)$, if the transport voltage V exceeds a certain crossover scale V^*. This scaling indicates anomalous inelastic relaxation processes to be present. It is demonstrated that the latter can be induced by quantum impurities with a degenerate internal degree of freedom, i.e. by Kondo impurities. We review a perturbative renormalization group method to describe the Kondo effect in an arbitrary stationary non-equilibrium situation as well as in a magnetic field, and show that the experiments are explained in detail by a very low concentration of Kondo impurities, with $V^* \approx T_K$, the Kondo temperature. It is discussed how this provides a possible explanation of the observed low-temperature plateau of the decoherence time in mesoscopic conductors.

Recent years have witnessed a tremendous development in the fabrication techniques of nanoscopic devices as well as in the preparation and control of quantum states in electronic systems. The functionality of such devices relies usually on the coherence of the quantum states involved. However, it has been recognized that in disordered, metallic quantum wires the phase coherence time τ_φ, as extracted from the weak localization correction to the magnetoresistance, seems to saturate for low temperatures T at a plateau value [1] rather than diverge as $\tau_\varphi \sim T^{-2/3}$, as expected from the theory of one-dimensional diffusive electron systems with Coulomb interaction [2,3]. It can be of crucial importance for the design of quantum coherent electron devices to uncover the microscopic origin of the underlying dephasing processes. Roughly at the same time when the notion of the anomalous dephasing time plateau was introduced as a physical effect, it was demonstrated by the Saclay group that a detailed investigation of the quasiparticle distribution function f in a quantum device in stationary non–equilibrium is an exceedingly sensitive tool to investigate the interactions at work [4]. This is because the shape of the distribution f, as compared to a non–interacting system, is only influenced by inelastic, i.e. dephasing processes, while linear response measurements like the conductivity are in general hampered by a large elastic impurity scattering background.

In this article we review the theoretical analysis of direct measurents of the non–equilibrium distribution function in resistive metallic nanowires. The experiments on Au and Cu wires [4,5] reveal a scaling property of the distribution function at a position x along the wire in terms of the quasi-particle energy E and the applied transport voltage V, $f_x(E,V) = f_x(E/V)$ for voltages exceeding a crossover scale V^*. As will be shown, the origin of this behavior can be traced back to an extremely low concentration (a few ppm) of Kondo impurities present in these nanowires, with $V^* \approx T_K$, the Kondo temperature. For a general introduction to the physics of Kondo impurities see Refs. [6,7]. On the other hand, samples of even greater nominal purity [5] as well as Ag wires [8] do not show the low-temperature anomaly. To calculate the distribution function theoretically, we have previously used the so–called Non–Crossing Approximation (NCA) for Anderson impurities in the Kondo regime out of equilibrium [9,10]. The theory allows for a quantitative comparison with experiment. In fact, after determining T_K as the energy scale V^* where deviations in the distribution function from the scaling form occur, the concentration of Kondo impurities in the wire is the only adjustable parameter of the theory and, hence, can be extracted by fitting the theoretical curves to experiment. Comparing the impurity concentration determined in this way to the low–temperature plateau values of the dephasing time in several similarly fabricated nanowires strongly suggests that the dephasing time plateau is also caused by Kondo scattering [5,9,10]. Very recently there have been experimental studies probing especially the question whether these impurities are of magnetic or of non–magnetic nature. These experiments investigated (1) the non–equilibrium distribution in an external magnetic field B [11] and (2) the decoherence time in wires with deliberately added magnetic impurities [12]. The results strongly suggest that the Kondo impurities are of magnetic origin.

As for the theoretical description, it is well known that the NCA fails in the presence of a magnetic field. Theoretical studies for $B \neq 0$ have been done in Ref. [13], but do not permit a satisfactory comparison with experiments. Therefore, we will use here the concept of a perturbative coupling constant renormalization group (RG) method, recently developed by us [14] to describe a single Kondo defect or quantum dot in stationary non–equilibrium, i.e. coupled to two electron reservoirs at different chemical potentials, with or without an external magnetic field. We outline its generalization to the present case of a quantum wire with a finite concentration of Kondo impurities and an arbitrary, position dependent non–equilibrium distribution function. In this case, the quantum Boltzmann equation for $f_x(E,V)$ must be solved along with the RG equations. Alternatively, we use in the case of vanishing magnetic field an auxiliary particle representation of the Anderson impurity model in the Kondo regime [6], using the so–called Non–Crossing

Approximation (NCA) in non–equilibrium [9], and show corresponding numerical results.

The paper is organized as follows. In section 1 we recollect the experimental setup to measure the non–equilibrium distribution function in a metallic wire, along with the most important experimental results. The perturbative RG method for a Kondo defect away from equilibrium and its generalization to a finite defect concentration and arbitrary distribution function are outlined in section 2. The quantitative fits of the Kondo-induced distribution functions calculated within NCA to the experimental results are shown in section 3. Here we also discuss the connection to related measurements of the decoherence time in quantum wires fabricated in a similar way, before some conclusions are drawn in section 4

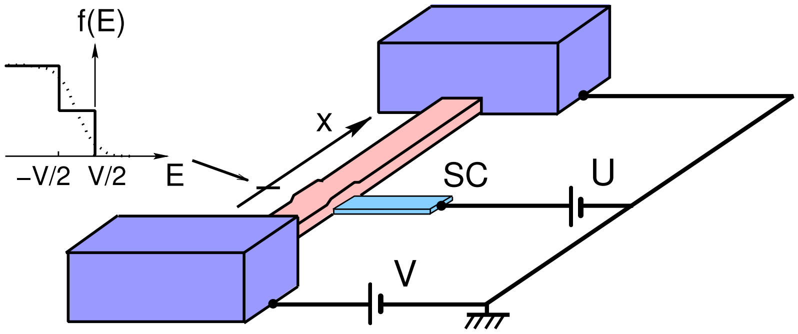

Fig. 1. Experimental setup for measuring the non-equilibrium distribution function in a metallic nano-wire. Also sketched is the distribution function as measured at position x along the wire, and compared to the hot–electron case.

1 Experimental Method and Results

The distribution function $f(E, U)$ was measured [4,8] in a three–terminal setup where a non–equilibrium current was driven through a Cu, Au or Ag nano-wire contacted by two reservoirs at chemical potentials $\mu_{L,R} = \pm V/2$, respectively (Fig. 1). (Throughout we will use units such that $\hbar = 1$, $k_B = 1$, $e = 1$, and $\mu = 1$, where e and μ are the elementary charge and the magnetic moment, respectively). In addition, a superconducting Al tunneling junction was attached at a position x along the wire, the Al slab being in equilibrium with itself. For a voltage U across the junction the tunneling current is given by

$$j_{tunnel} = \frac{e}{\hbar}|t|^2 \sum_{\sigma} \int dE \big[f(E) - f^o(E+U)\big] N_0 \; N_{sc}(E+U) \; , \qquad (1)$$

where t is the (assumed energy independent) tunneling matrix element, $f^o(E)$ is the Fermi distribution function in the superconductor, and N_0, N_{sc} denote the density of states in the wire and in the superconductor, respectively. Since for the voltages used in the experiment N_0 is flat and the BCS density of states N_{sc} is measured independently, the pronounced peak structure of $N_{sc}(E)$ can be used to deconvolute Eq. (1) to obtain the non–equilibrium distribution $f_x(E,V)$ at position x in the wire. Measurements in magnetic field were also performed, exploiting the Al'tshuler-Aronov dip [3] in the density of states of a strongly disordered (instead of superconducting) slab of the tunneling contact in order to deconvolute Eq. 1. The electronic transport in the wire is diffusive with diffusion coefficient $\mathcal{D}$. Length L and thickness d of the wire are such that the diffusion modes are one–dimensional, but the Fermi surface is three–dimensional, and the Coulomb and phonon scattering times are large compared to the electronic diffusion time $\tau_D = L^2/D$ through the wire, so that equilibration due to these processes can be neglected [4]. In this situation, assuming purely elastic scattering, one expects the distribution function at a given position x to be a linear superposition of the Fermi functions in the reservoirs,

$$f_x^{elast}(E,V) = \big(1-\frac{x}{L}\big) f^o(E+V) + \frac{x}{L} f^o(E) \ , \tag{2}$$

Eq. (2) is a solution of the diffusive Boltzmann equation (7) [15], for vanishing collision integral (see section 2.1). This situation is to be distinguished from the hot electron regime, where local equilibration occurs due to inelastic processes [16].

Typical measured distribution functions are shown in Fig. 2a). They exhibit rounding of the Fermi steps as compared to Eq. (2) and obey scale invariance with respect to the transport voltage V, $f(E,V) = f(E/V)$, when V exceeds a certain low energy scale, $V \gtrsim 0.1$ meV (Fig. 2b)) [4]. Deviations from scaling were observed again for voltages larger than a high energy scale $E_o \simeq 0.5$ meV. The latter may be explained by reservoir heating effects or by the electrons coupling to additional degrees of freedom at high energies.

By a heuristic argument [4], the origin of the observed scaling behavior can be traced back to an anomalous electron–electron interaction $\tilde{v}(\varepsilon)$ which scales with the energy transfer ε as $\tilde{v}(\omega) \propto 1/\varepsilon$. We reproduce this argument here: The scaling property implies that the equation of motion for $f(E,V)$, the Boltzmann equation and, as a consequence, the inelastic single–particle collision rate $1/\tau(E)$ are scale invariant. Assuming for the moment that $\tilde{v}(\varepsilon)$ has no essential momentum dependence, $1/\tau$ is given in 2nd order perturbation theory (PT) by

$$\frac{1}{\tau(E)} \equiv \frac{1}{\tau(E/V)} \simeq N_0^3 \int d\varepsilon \int d\varepsilon' |\tilde{v}(\varepsilon)|^2 \tilde{F}\Big(\frac{E}{V}, \frac{\varepsilon}{V}, \frac{\varepsilon'}{V}\Big) \ . \tag{3}$$

Here $\tilde{F}$ is a combination of distribution functions f guaranteeing that there is only scattering from an occupied into an unoccupied state. Therefore, the

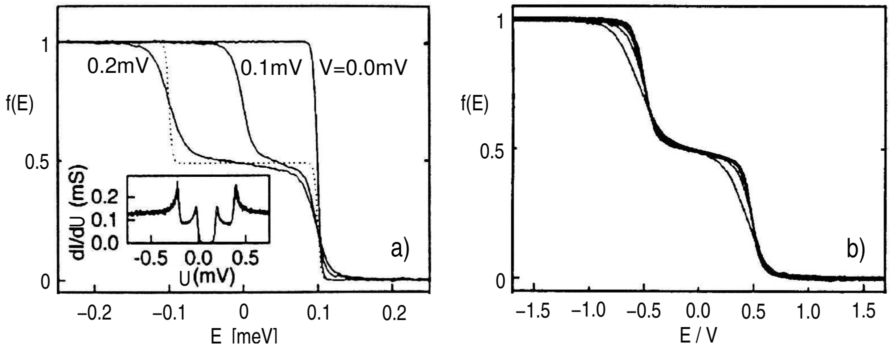

Fig. 2. a) Typical measured distribution functions in the middle of a resistive Cu wire (L = 1.5 μm,x/L=0.5), bath temperature T = 25 mK. The distribution for the non–interacting case is shown as a dashed line. The inset shows raw data for the differential conductance of the tunnel junction, exhibiting the BCS singularities at the gap edges. b) Scaling collapse of the data of a). (Data reproduced from Ref. [4]).

experimental results about the scaling property of f imply that $\tilde{F}$ depends only on the dimensionless energies as displayed in Eq. (3). Demanding scale invariance for $1/\tau$ with respect to V, i.e. making the frequency integrals dimensionless, implies a characteristic energy dependence of the interaction and the single particle scattering rate, $\tilde{v}(\varepsilon) \propto 1/\varepsilon$. Extrapolated to the Fermi energy E_F this means that in 2nd order perturbation theory (PT) the resulting quasiparticle relaxation rate would not vanish at E_F [4]. One can, therefore, conjecture that the anomalous scaling form $f_x(E/V)$ and the apparent low-temperature saturation of the dephasing time observed [1] in the magnetoresistance $\rho(B)$ of nanowires might have the same microscopic origin. To substantiate this speculation, a quantitative calculation in non-equilibrium is needed, and the perturbative infrared singularity of $\tilde{v}(\varepsilon)$ signals that one has to re-sum infinite orders of PT.

We note that the momentum depedence of the dynamically screened Coulomb interaction in a disordered system, induced by the one–dimensional diffusion mode of the nanowire, [3] would not be consistent with the observed scaling property, and that other collective modes which could induce a singular effective interaction, are not expected in the simple Au or Cu wires. We conclude that, within the 2nd order PT argument discussed above, $\tilde{v}(\varepsilon) \propto 1/\varepsilon$, i.e. has no essential momentum dependence and should thus be of local origin. Anomalous low–energy behavior of local origin can be induced by the Fermi surface singularities characteristic for Kondo type systems. [6,7]. Based on such considerations, Kondo impurities have been proposed as the origin of the anomalous energy relaxation [9,10,17,18]. Kaminski and Glazman showed first that Kondo scattering can induce an effective local electron-electron interaction consistent with the observed scaling, including the deviations thereof [17]. Inelastic scattering by Kondo impurities has also been considered in Ref. [19].

2 Theory

2.1 The Model and General Relations

We now turn to the calculation of the distribution function $f_x(E,V)$ in a wire with uniform concentration of Kondo impurities and with a finite bias V applied between its ends. For a single Kondo impurity embedded in the electron sea we consider the Kondo Hamiltonian

$$H = \sum_{\mathbf{k},\sigma} \varepsilon_{\mathbf{k}} c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} - BS_z + \sum_{\mathbf{k},\mathbf{k}',\sigma,\sigma'} J\,\mathbf{S}\cdot(c^\dagger_{\mathbf{k}'\sigma'}\tau_{\sigma'\sigma}c_{\mathbf{k}\sigma}), \tag{4}$$

where the first term describes the conduction electron sea with the flat bare density of states N_0 and the band width D, and $J>0$ is the antiferromagnetic spin coupling. It is convenient to use the pseudofermion representation of the local spin 1/2 [20,6]

$$\mathbf{S} = \frac{1}{2}\sum_{\sigma,\sigma'} f^\dagger_{\sigma'}\tau_{\sigma'\sigma}f_\sigma . \tag{5}$$

Here τ is the vector of Pauli matrices, and $f^\dagger_\sigma$, f_σ are pseudofermion operators obeying the constraint $Q=\sum_\sigma f^\dagger_\sigma f_\sigma = 1$, which can be implemented exactly (for details of this technique see, e.g., Refs. [21,22]).

How can scattering from a Kondo defect, which has only a *degenerate* internal degree of freedom, lead to inelastic processes and hence to a redistribution of the occupation of states in the wire? The physical origin is the finite decay rate of the internal degree of freedom. It is given by the transverse spin relaxation rate Γ (see Eq. (11) below) and is in the large bias limit, $\ln(V/T_K)\gg 1$ (but $\ln(V/D)\ll 1$), proportional to V,

$$\Gamma_0(V) = 2\pi\frac{x}{L}\Big(1-\frac{x}{L}\Big)(N_0J)^2 V\ , \tag{6}$$

analogous to the Korringa relaxation rate, with T replaced by V [9]. Note that towards smaller voltages there is a logarithmic renormalization $N_0J\to 1/[2\ln(V/T_K)]$, with $T_K = D\sqrt{N_0J}e^{-1/2N_0J}$. The resulting life–time broadening of the local states gives ultimately rise to inelastic scattering with energy exchange $\sim 1/\tau_s(V)$. Hence, it is, roughly speaking, the approximate proportionality Eq. (6) that induces a broadening of the Fermi steps $\propto V$ (up to log corrections), i.e. the observed scaling property.

An accurate description of these non-equilibrium phenomena requires a detailed, quantitative calculation of the distribution function. We consider $f_x(E,V)$ in a resistive nanowire of length L, subject to the boundary conditions that the left ($x=0$) and the right ($x=L$) leads are in equilibrium at their respective chemical potentials, i.e. $f_{x=0}(E,V)=f^o(E-V/2)$,

$f_{x=L}(E,V) = f^o(E+V/2)$, with $f^o(E) = 1/(\mathrm{e}^{E/T}+1)$ the Fermi distribution. The lesser ($<$) and the greater ($>$) conduction electron Keldysh Green's functions read $G_x^<(\mathbf{k},E) = -2\pi i f_x(\mathbf{k})\,\mathrm{Im}G_x^r(\mathbf{k},E)$ and $G_x^>(\mathbf{k},E) = 2\pi i[1-f_x(\mathbf{k})]\,\mathrm{Im}G_x^r(\mathbf{k},E)$, respectively, where E, $\mathbf{k}$ denote quasiparticle energy and momentum [23]. A superscript ^{r} indicates a retarded propagator. The distribution function is determined by the stationary quantum Boltzmann equation, which in a disordered electron system with diffusion constant $\mathcal{D}$ takes the form [15]

$$-\mathcal{D}\nabla_x^2 f_x(E,U) = \mathcal{I}\{f_x(E,U)\} \ . \tag{7}$$

The collision integral

$$\mathcal{I} = \frac{1}{2\pi N_0}\sum_p [\Sigma_x^<(E)G_x^>(\mathbf{p},E) - \Sigma_x^>(E)G_x^<(\mathbf{p},E)] \ . \tag{8}$$

is expressed in terms of the selfenergies $\Sigma^{\lessgtr}$ for scattering into ($<$) and out of ($>$) states with given energy E. For low concentration of Kondo impurities c_{imp} the selfenergies are proportional to the single–electron T-matrix of a single impurity, $\Sigma_x^{\lessgtr}(E) = c_{imp}t_x^{\lessgtr}(E)$. We emphasize that, apart from the assumption of small c_{imp}, the formulation Eqs. (7), (8) is exact once the T-matrix is known.

This leaves us with the necessity to calculate the collision integral (and other physical quantities) in a controlled way. As is well known [6] low order PT is not sufficient for the Kondo problem because of its infrared divergencies. This remains true even in non–equilibrium, where the divergencies are cut off by inelastic terms like the transverse spin relaxation rate Γ, since the terms of PT are logarithmic and the convergence of the perturbation series is slow. Hence, a resummation to infinite order is required. Such resummations have been performed by several authors, using the log resummation scheme in equilibrium for the energy dependence of the collision kernel [17], using a factorization of the collision kernel and the Nagaosa resummation [18], or employing an NCA calculation of the collision integral Eq. (8) in non-equilibrium [9]. However, these resummations are either technically cumbersome or use an uncontrolled selection of terms in the resummation of PT. Notably, the NCA is known to give good quantitative results for vanishing magnetic field B in the energy range above and down to about 0.1 T_K, but completely fails for $B \neq 0$. For a discussion of the origin of this failure see [24]. Therefore, we here propose a poor man's scaling method for general stationary non-equilibrium situations which is technically simple to perform and at the same time allows for a systematic resummation of the terms of PT.

2.2 Poor Man's Scaling Method in Non–equilibrium

The coupling constant renormalization group method relies on the notion of universality, i.e. the fact that physical quantities may be expressed solely in terms of the effective low–energy scale of the problem (e.g. T_K), and do not explicitly depend on the microscopic parameters of the underlying Hamiltonian, in particular not on the band cutoff D. As a consequence, it is possible to absorb a change of the D into a renormalization of the coupling constants by the requirement that the physically observable quantities, like the T–matrix, (or their irreducible parts) be invariant under this cutoff rescaling. In the semianalytic approach considered here it is only possible to implement the invariance under an infinitesimal cutoff rescaling δD up to a given, finite order n of PT, typically $n = 2$ (1–loop RG). This leads to the perturbative flow equations for the renormalized coupling constants. The fact that the invariance of physical quantities has been implemented to nth order PT (see above) dictates for consistency that the irreducible part of any such quantity must now be calculated from the renormalized coupling constants in that same nth order PT. The latter statement is crucial especially for the RG in non–equilibrium, since here the RG equations themselves will not only involve the coupling constants (or coupling functions) but also physically observable quantities, namely the transverse spin relaxation rate Γ and the quasiparticle distribution function, which, in general, must be determined selfconsistently by solving the Boltzmann equation.

To 2nd order PT, the coupling function renormalizations are due to the vertex corrections shown in Fig. 3a), whose real parts are logarithmic. The intermediate electron lines in Fig. 3a) carry the distribution function $f_x(E,V)$ inside the wire. Although the Fermi steps in $f_x(E,V)$ will be smeared due to inelastic relaxation, we still expect, at least for small c_{imp}, the dominant contributions to come from the Fermi energies $\mu_{L,R} = \pm V/2$ in the left (L) or right (R) lead. Therefore, we choose the running cutoffs symmetric with respect to $\mu_{L,R}$ [14]. We define dimensionless coupling functions $g(\omega) = N_0 J(\omega)$, where ω is the energy of the scattering electron. The spin structure of these coupling functions is given by two invariant amplitudes, $\tilde{g}_\perp$ for spin flip and $\tilde{g}_{||\sigma}$ for spin non-flip processes:

$$g^{\alpha\sigma,\omega;\alpha'\bar{\sigma},\omega-\gamma B}_{\gamma,-\gamma B/2;\bar{\gamma},B/2} = (\tau^x_{\gamma\bar{\gamma}}\tau^x_{\sigma\bar{\sigma}} + \tau^y_{\gamma\bar{\gamma}}\tau^y_{\sigma\bar{\sigma}})\,\tilde{g}_\perp(\omega - \gamma B/2) \tag{9a}$$

$$g^{\alpha\sigma,\omega;\alpha\sigma,\omega}_{\gamma,-\gamma B/2;\gamma,-\gamma B/2} = \tau^z_{\alpha\alpha}\tau^z_{\gamma\gamma}\tilde{g}_{||\sigma}(\omega)\ , \tag{9b}$$

where $g^{a;a'}_{b;b'}$ denotes the coupling function for conduction electrons of spin σ and energy ω in lead $\alpha(a = (\alpha,\sigma,\omega))$ interacting with a pseudo fermion in state $b = (\gamma,\omega_f)$ and going into states a', b'. Using furthermore the fact that the pseudofermion propagators are sharply peaked at the Zeeman energies $\pm B$, one obtains the RG equations for the dimensionless, energy dependent coupling functions at position x along the wire,

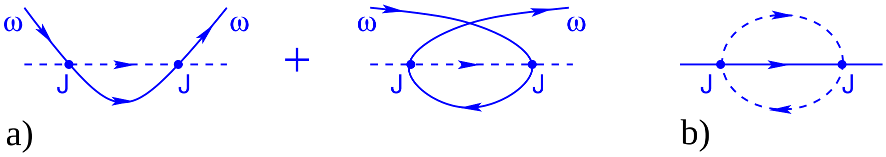

Fig. 3. a) Diagrams generating the 2nd order renormalization of the conduction electron–local spin vertex. Solid and dashed lines denote the electron and the pseudofermion propagators, respectively. b) 2nd order expression for the conduction electron T–matrix.

$$\frac{\partial \tilde{g}_{x||\sigma}(\omega)}{\partial \ln D} = -\frac{1}{4} \sum_{\alpha,\beta=-1,1} \tilde{g}_{x\perp}(\frac{B+\beta V}{2})^2 \times \tag{10a}$$
$$\Big[2f_x(\alpha D + \beta V/2, V) - 1\Big]\, \tilde{G}^f\Big[\frac{\omega + \sigma(B + \beta\frac{V}{2})}{\alpha D}\Big]$$
$$\frac{\partial \tilde{g}_{x\perp}(\omega)}{\partial \ln D} = -\frac{1}{2} \sum_{\alpha,\beta,\sigma=-1,1} \tilde{g}_{x\perp}(\frac{\sigma B+\beta V}{2})\tilde{g}_{x||\sigma}(\frac{\beta V}{2}) \times \tag{10b}$$
$$\Big[2f_x(\alpha D + \beta V/2, V) - 1\Big]\, \tilde{G}^f\Big[\frac{\omega + \frac{\sigma B+\beta V}{2}}{\alpha D}\Big]\ ,$$

where $\tilde{G}^f(y) = 1/(1+y)$ is related to the real part of the pseudofermion propagator, and the initial conditions are $\tilde{g}_{x||\sigma}(\omega) = \tilde{g}_{x\perp}(\omega) = N_0 J$. Note that for the case of a single impurity coupled to two leads at different chemical potentials (no x dependence), the statistical factors and the functions $\tilde{G}_f$ in Eqs. (10a), (10b) can effectively be combined to threshold functions, so that the above equations reduce to the poor man's scaling equations derived in Ref. [14].

In the RG equations (10a), (10b) it is up to now assumed that the relaxation rate Γ of the impurity spin state destroying quantum coherence is negligible compared to V or B. However, due to the finite current, relaxation processes contributing to Γ are present even at $T = 0$, and lead to an imaginary part of the self-energy of the pseudofermions (e.g. in the PF propagator in the diagrams of Fig. 3a) and to vertex corrections. We identify Γ with the transverse spin relaxation rate [14], which is in 2nd order in the renormalized couplings given by

$$\Gamma_x = \frac{\pi}{4\hbar} \sum_{\gamma=-1,1} \int d\omega \Big[\tilde{g}_{x||\gamma}(\omega)^2 f_x(\omega, V)[1 - f_x(\omega, V)] \tag{11}$$
$$+\tilde{g}_{x\perp}(\omega - \gamma B/2)^2 f_x(\omega, V)[1 - f_x(\omega - \gamma B, V)]\Big].$$

To incorporate the effect of decoherence in the RG equations, the pseudofermion propagators are broadened by the rate Γ, i.e. we re–define $\tilde{G}^f(y)$ as $\tilde{G}^f_{\Gamma/\alpha D}(y) = (1+y)/[(1+y)^2 + (\Gamma/\alpha D)^2]$.

As a further complication, the RG equations (10a), (10b) contain the as yet unknown distribution functions f_x at the positions x in the wire. They are to be determined by solving the Boltzmann equation (7), where the T–matrix entering the collision integral (8) must be calculated in 2nd order PT in the renormalized coupling constants, see the discussion at the beginning of this section. The corresponding expression for $t_x^{\lessgtr}(E)$ is depicted diagrammatically in Fig. 3b). This means that the Boltzmann equation (7), with Eqs. (8), and the set of RG equations (10a)(10b), with the spin relaxation rate Eq. (11) must be solved selfconsistently. The solution of this set of equations is in progress. It should allow for a well controlled description of the distribution function in non-equilibrium quantum wires containing a finite concentration of Kondo impurities, both with and without magnetic field B. In the following section we show results obtained from the NCA solution of this problem for $B = 0$ [9].

To conclude this section, we analyze the scaling properties of $f_x(E, V)$ following from the RG approach for zero magnetic field B. From the collision integral Eq. (8) and the T–matrix (Fig. 3b)) it follows that $f_x(E, V)$ has the scaling property $f_x(E, V) = f_x(E/V)$ exactly if the coupling functions depend on the energy ω only through ω/V. By defining a dimensionless cutoff D/V it is seen that all energies in the RG equations become dimensionless (in units of V), provided that (1) the distribution function depends only on the dimensionless energy E/V and (2) the spin relaxation rate Γ is proportional to V (Korringa behavior). The latter condition is obeyed in the large bias limit, $\ln(V/T_K) \gg 1$, with logarithmic corrections for smaller bias. Hence, observing that the initial conditions $\tilde{g}_{x||\sigma}(\omega) = \tilde{g}_{x\perp}(\omega) = N_0 J = const(\omega)$ trivially fulfill the condition that the coupling constants depend only on the dimensionless energy, the selfconsistent solution $f_x(E/V)$ is scale invariant for large bias, however with sizeable logarithmic corrections in a wide voltage range, as the bias V approaches T_K from above. This is in agreement with the experimental findings, if one identifies the crossover scale for deviations from scaling $V^* \approx T_K$. For finite magnetic field the scaling property naturally breaks down, as seen from the RG equations.

3 Comparison with experiment: NCA solution

The numerical NCA solutions show approximate scaling within a factor of 4 to 9 in V, depending on parameters. This fact, that scaling is obeyed only within a finite, transient voltage range, is because of the logarithmic corrections to the linear in V behavior of the spin relaxation rate Γ, Eq. (6), and of vertex corrections entering the NCA equations (or the RG equations, respectively), see the discussion at the end of section 2.2. For $V \lesssim 10V^*$ approximate scaling cannot be esablished anymore. This provides for $T \ll T_K$ a rough estimate, and for $T > T_K$ an upper bound on T_K. The experimental parameters [4,8] are in the regime $T \lesssim T_K \ll V$.

For the numerical evaluations we assume magnetic impurities and take $T_K \approx$ 0.1 K in Cu and $T_K \approx 0.5$ K in Au wires (corresponding to $N_0 J = 0.041$

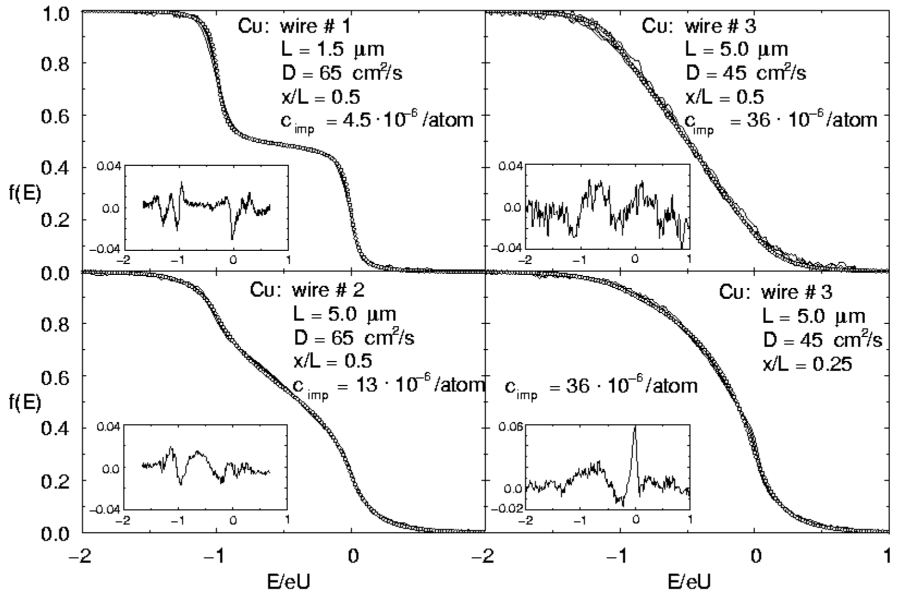

Fig. 4. Non–equilibrium distribution functions for three different Cu samples. Black lines: experimental results; [4]. Open circles: theory for $V \gg T_K$. Deviations from scaling at smaller V [1] are also reproduced by the theory (not shown). The fitted c_{imp} values are indicated. The insets show the difference between the experimental and the theoretical curves. For sample 3 $f_x(E/V)$ was measured simultaneously at two different positions x; hence, both curves are fitted by one single value of c_{imp}.

and $N_0 J = 0.048$, respectively). These values are consistent with the experimentally observed scale for deviations from scaling and with independent estimates of T_K for these samples [8]. After T_K is fixed, c_{imp} is the only adjustable parameter of the theory. The results for $f_x(E,V)$, as measured by a tunnel junction attached to the wire, are shown in Figs. 4, 5. Excellent quantitative agreement with experiments [4,8] is obtained for all samples. In Au wires the fitted values of c_{imp} are consistent with (although somewhat higher than) independent estimates of the magnetic impurity concentration [8], considering the roughness of both estimates. This suggests that the scaling behavior of $f_x(E,V)$ in the Au samples is due to magnetic impurities. Furthermore, in all Cu samples the fitted c_{imp} is $\sim 10^2$ times smaller than in Au. This systematics is in accordance with c_{imp} estimated from the plateau in the T dependence of the dephasing time τ_φ in similarly prepared samples [8,25].

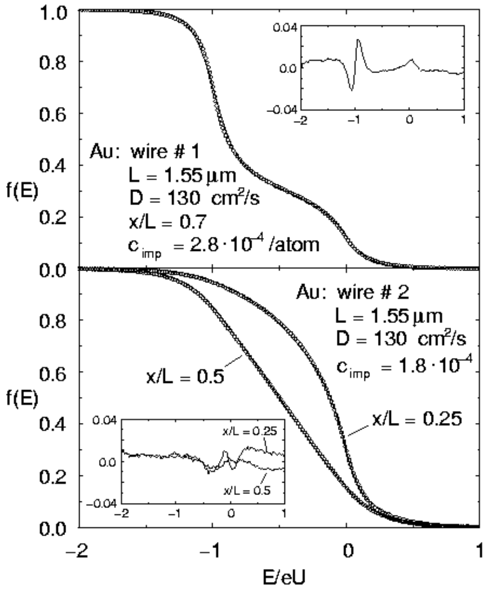

Fig. 5. The same quantities as in Fig. 4 are shown, however for Au wires. Experimental results are reproduced from [8].

4 Conclusion

We have described a recently introduced poor man's scaling method (perturbative coupling constant RG on 1–loop level), applicable to Kondo systems in non-equilibrium. In non-equilibrium the renormalized couplings necessarily acquire an ω–dependence, and decoherence processes are incorporated by including the transverse spin relaxation rate Γ in the RG equations. We propose a generalization to the case of an arbitrary, stationary non-equilibrium distribution in the electron sea. Such a situation is realized, e.g. in quantum wires with a finite concentration of Kondo impurities, subject to a large bias voltage. In this case, the distribution function, depending on the quasiparticle energy E, the bias voltage V, and on the position x, enters the RG equations and must be determined selfconsistently during the RG procedure by solving the respective quantum Boltzmann equation.

We have shown that the quasiparticle distribution in a quantum wire with Kondo impurities, determined by this set of coupled equations, obeys the scaling property $f_x(E,V) = f_x(E/V)$ in the regime of exponentially large bias, $\ln(V/T_K) \gg 1$, with deviations from scaling for smaller voltage, in agreement with related experiments on Cu and Au quantum wires. Nonequilibrium solutions of this problem within the NCA for vanishing magnetic field give quantitative agreement with related experiments and allow to extract the concentration of Kondo impurities from fitting the theoretical curves to the experimental data. A detailed comparison of these results with the dephasing time measurements in similarly prepared samples strongly suggests that the anomalous low–temperature plateau of τ_φ in Cu and Au wires is caused by Kondo impurities. A solution of the non-equilibrium RG equations for resistive wires in magnetic field is in progress.

We thank A. Zawadowski, S. De Franceschi, L.I. Glazman, J. König, O. Parcollet, H. Pothier, H. Schoeller and G. Sellier for helpful discussions. This work was supported in part by the DFG Forschungszentrum for Functional Nanostructures (CFN) and by the Emmy Noether program (A.R.) of the DFG.

References

1. P. Mohanty, E.M.Q. Jariwala and R. A. Webb, Phys. Rev. Lett. **78**, 3366 (1997).
2. B. L. Al'tshuler, A. G. Aronov, and D. E. Khmelnitskii, J. Phys. C: *Solid State Physics* **15**, 7367 (1982).
3. For a review see B. L. Al'tshuler and A. G. Aronov, in *Electron–Electron Interactions in Disordered Systems*, 1 (North–Holland, Amsterdam, 1985).
4. H. Pothier, S. Guéron, Norman. O. Birge, D. Esteve, and M. H. Devoret, Phys. Rev. Lett. **79**, 3490 (1997).
5. F. Pierre, H. Pothier, D. Esteve, M.H. Devoret, A.B. Gougam, and N.O. Birge in *Kondo Effect and Dephasing in Low–Dimensional Metallic Systems* V. Chandrasekhar, C. van Haesendonck, and A. Zawadowski eds., Nato Science Series II, **50**, 119 (Kluwer, 2001).
6. For a comprehensive introduction to the Kondo and related problems see A. C. Hewson, *The Kondo Problem to Heavy Fermions*, (Cambridge University Press, 1993).
7. D. L. Cox and A. Zawadowski, Adv. Phys. **47**, 599 (1998).
8. F. Pierre, H. Pothier, D. Esteve, and M.H. Devoret, J. Low Temp. Phys. 118, 437-445 (2000).
9. J. Kroha and A. Zawadowski, Phys. Rev. Lett. **88**, 176803 (2002).
10. J. Kroha, Adv. Solid State Phys. **40**, 216 (2000); J. Kroha, in *Kondo Effect and dephasing in low-dimensional metallic systems*, V. Chandrasekhar, C. v. Haesendonck, and A. Zawadowski, eds., NATO Science Series II **50** , 133 (Kluwer, 2001).
11. A. Anthore, F. Pierre, H. Pothier, and D. Esteve, Phys. Rev. Lett. **90**, 076806 (2003).
12. F. Schopf, C. Bäuerle, W. Rabaud, and L. Saminadayar, Phys. Rev. Lett. **90**, 056801 (2003).
13. G. Göppert, Y. M. Galperin, B. L. Altshuler, and H. Grabert, Phys. Rev. B **66**, 195328 (2002).
14. A. Rosch, J. Paaske, J. Kroha, and P Wölfle, Phys. Rev. Lett. **90**, 076804 (2003).
15. K. E. Nagaev, Phys. Lett. A **169** 103 (1992); Phys. Rev. B **52**, 4740 (1995).
16. Kozub and Rudin, Phys. Rev. B **52**, 7853 (1995).
17. A. Kaminski and L.I. Glazman, Phys. Rev. Lett. **86**, 2400 (2001).
18. G. Göppert, H. Grabert, Phys. Rev. B **64**, 033301 (2001).
19. J. Sólyom, A. Zawadowski, Z. Phys. B **226**, 116 (1996).

20. A. A. Abrikosov, Physics **2** 5 (1965).
21. A review can be found in N. E. Bickers, Rev. Mod. Phys. **59**, 845 (1987).
22. T. A. Costi, J. Kroha, and P. Wölfle, Phys. Rev. B **53**, 1850 (1996).
23. J. Rammer and H. Smith, Rev. Mod. Phys. **58**, 323 (1986).
24. S. Kirchner and J. Kroha, J. Low Temp. Phys. **126**, 1233 (2002).
25. A. B. Gougam, F. Pierre, H. Pothier, D. Esteve, and N.O. Birge, J. Low Temp. Phys. **118**, 447 (2000).