Dissipatively driven waveguides

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Ich versichere, dass ich diese Arbeit selbstständig verfasst und keine anderen als die angegebenen Quellen und Hilfsmittel benutzt sowie die Zitate kenntlich gemacht habe.

Bonn, ...

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Contents

1 Introduction

In contrast to theoretical models of closed quantum systems, experimental realisations always underlie dissipative effects. Actually, many experiments are designed to observe the behaviour of systems governed by exactly these effects. For the theoretical description one works with open quantum systems having a non-unitary time-evolution specified by so called master equations. Trying to characterise and understand open quantum systems leads, in fact, to many new interesting quantities and effects. One example is the quantum Zeno effect, experimentally observed e.g. by Barontini (2013), where a localised dissipation of particles from a Bose-Einstein condensate was realised by using an electron beam.

Our open quantum system of interest, and thus, the main topic of this thesis are dissipatively driven waveguides. In this case, the dissipation is given by the loss of photons during the time where the light evolves through the waveguides. For our application, we focus on the Lindblad master equation, which we introduce at the beginning after a short revision about quantum systems in general. Additionally, we describe the system with a non-hermitian Hamiltonian and compare both results.

Starting with a homogeneous system where the hopping amplitude and the dissipation strength are the same for each waveguide, we obtain with both formalisms a solution for arbitrary numbers of waveguides and particles for a time-independent as well as a time-dependent dissipation strength. After that, we focus on a simpler model containing only two waveguides and one particle. The significant feature here is that only one of these two waveguides has the property of losing photons. This leads to the appearance of the quantum Zeno effect. Moreover, we obtain in the numerical solutions from the Lindblad equation for a time periodic dissipation strength resonances at specific frequencies. These occur also in the solution with the non-hermitian Hamiltonian, which we get by using Floquet theory.

2 Theory of quantum systems

As a starting point for our description of quantum systems, we give a short introduction to the density matrix formalism and comment on the distinction between closed and open systems. This leads to the Lindblad master equation as an effective equation for open systems. For a general introduction to quantum mechanics, we refer to Shankar (2012). Regarding the description of open quantum systems, we mainly follow Breuer and Petruccione (2002).

2.1 Closed quantum systems

The Schrödinger equation describes a closed quantum mechanical system by

$$i\hbar \frac{\mathrm{d}}{\mathrm{d}t} |\Psi(t)\rangle = H(t) |\Psi(t)\rangle$$

with the state vector $|\Psi(t)\rangle$ in a Hilbert Space $\mathcal{H}$ and the Hamiltonian H. Due to the linearity of this equation, a general solution is given in terms of the unitary time-evolution operator $U(t, t_0)$ as

$$|\Psi(t)\rangle = U(t, t_0) |\Psi(t_0)\rangle .$$

For a time-dependent Hamiltonian, the time-evolution operator can be written as a time ordered exponential

$$U(t, t_0) = T \ \exp\left\{-i \int_{t_0}^{t} \mathrm{d}t' H(t')\right\},$$

which in the time-independent case reduces to

$$U(t, t_0) = e^{-iH(t-t_0)/\hbar}.$$

Another possible way to describe a quantum mechanical system includes the density operator ρ. This formulation additionally allows to consider mixed states. A pure state can be written as a linear combination of basis states $|\Phi_i\rangle$ of the Hilbert space

$$|\Psi(t)\rangle = \sum_i c_i |\Phi_i(t)\rangle$$

with complex coefficients c_i. In some basis, a mixed state can now be expressed by the density operator ρ given by

$$\rho(t) = \sum_i p_i |\Psi_i(t)\rangle \langle\Psi_i(t)|$$

where the coefficients p_i corresponds to the probability of being in the normalized state $|\Psi_i(t)\rangle$. The density operator ρ has to fulfil several properties in order to describe a physical state:

- The density operator has to be hermitian: $\rho^\dagger = \rho$
- The trace of the density operator has to be equal to one: $\mathrm{Tr}(\rho) = 1$
- The density operator has real[1] and positive eigenvalues

The equation of motion for the density operator follows from the Schrödinger equation and is called Liouville- von Neumann equation:

$$i\hbar \frac{\mathrm{d}}{\mathrm{d}t} \rho(t) = [H(t), \rho(t)]$$

[1]Note that this is already implied by the hermiticity of ρ.

In the density matrix formalism, the expectation value of an operator A can be calculated by

$$\langle A\rangle(t) = \mathrm{Tr}(\rho(t)A). \tag{2.1}$$

2.2 Open quantum systems

An open quantum mechanical system describes two coupled quantum Systems. One of them would be the system of our interest and the other one the environment. Both systems have their own Hilbert spaces, which are connected by the tensor product. Therefore, the Hilbert space of the total system can be written as

$$\mathcal{H}_{\mathrm{total}} = \mathcal{H} \otimes \mathcal{H}_{\mathrm{E}}.$$

Here $\mathcal{H}$ denotes the systems Hilbert space and $\mathcal{H}_{\mathrm{E}}$ the environments Hilbert space. Analogous the Hamiltonian $H_{\mathrm{total}}(t)$ for the total system can be decomposed out of the Hamiltonian for the quantum system and out of the Hamiltonian for the environment as

$$H_{\mathrm{total}}(t) = H(t) \otimes I_{\mathrm{E}} + I \otimes H_{\mathrm{E}}(t) + H_{\mathrm{I}}(t)$$

with the identity operator I. The additional Hamiltonian $H_{\mathrm{I}}(t)$ describes the interaction between the two systems. Due to this interaction, the dynamics of the system we are looking at are in general not unitary. Trying to solve this at all could get quickly complicated because of the huge amount of degrees of freedom of the environment. But under certain assumptions, one can find an approximate equation for the quantities we are interested in. This leads to the so called Master equations.

2.3 Lindblad master equation

For the system we are going to have a look on later, we will use the Lindblad master equation for the density matrix ρ on $\mathcal{H}$. This equation is similar to the Liouville-von Neumann equation but it contains an additional term, which is called the dissipator $\mathcal{D}(\rho(t))$:

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho(t) = -\frac{i}{\hbar}[H(t), \rho(t)] + \mathcal{D}(\rho(t))$$

The Dissipator includes the Lindblad or jump operators $\mathcal{L}$ of the system[2]

$$\mathcal{D}(\rho(t)) = \sum_i \gamma_i(t) \cdot (\mathcal{L}_i\rho(t)\mathcal{L}_i^\dagger - \frac{1}{2}\mathcal{L}_i^\dagger\mathcal{L}_i\rho(t) - \frac{1}{2}\rho(t)\mathcal{L}_i^\dagger\mathcal{L}_i)$$

with $\gamma_i \in \mathbb{R}_{\geq 0}$. The Lindblad equation describes a non-unitary time-evolution. Note that it preserves the hermiticity, the trace, and the positive semidefiniteness of ρ. Besides, it is a Markovian master equation, which means that one neglects memory effects in this case. One also says that the dynamics of our system of interest can be described by a quantum dynamical semigroup.

The Lindblad master equation can also be written down in the Heisenberg picture, in which the density matrix is constant. This leads to

$$\frac{\mathrm{d}}{\mathrm{d}t}A(t) = \frac{i}{\hbar}[H, A(t)] + \sum_i \gamma_i(t) \cdot (\mathcal{L}_i^\dagger A(t)\mathcal{L}_i - \frac{1}{2}\mathcal{L}_i^\dagger\mathcal{L}_i A(t) - \frac{1}{2}A(t)\mathcal{L}_i^\dagger\mathcal{L}_i) \tag{2.2}$$

for an arbitrary Schrödinger operator A with no explicit time dependence. It might be easier to just calculate the expectation values of the quantities of interest by solving the Lindblad equation in the Heisenberg picture for them than solving the equation for the whole density matrix. We are going to see this later.

[2]These are a basis of operators acting on the Hilbert space.

3 Dissipatively driven waveguides

The model we are going to use to describe the waveguides is depicted in figure 3.1. The photons propagate through the waveguides along the y-direction with the possibility of tunnelling to the nearest neighbour sites and dissipating out of the system. The propagation distance y corresponds to the time t, and thus, the time-dependence of the tunnelling amplitude and dissipation strength arises due to the spatial variations of the waveguides along the y-direction. An example for the experimental realisation can be found in the paper of Cherpakova (2018).

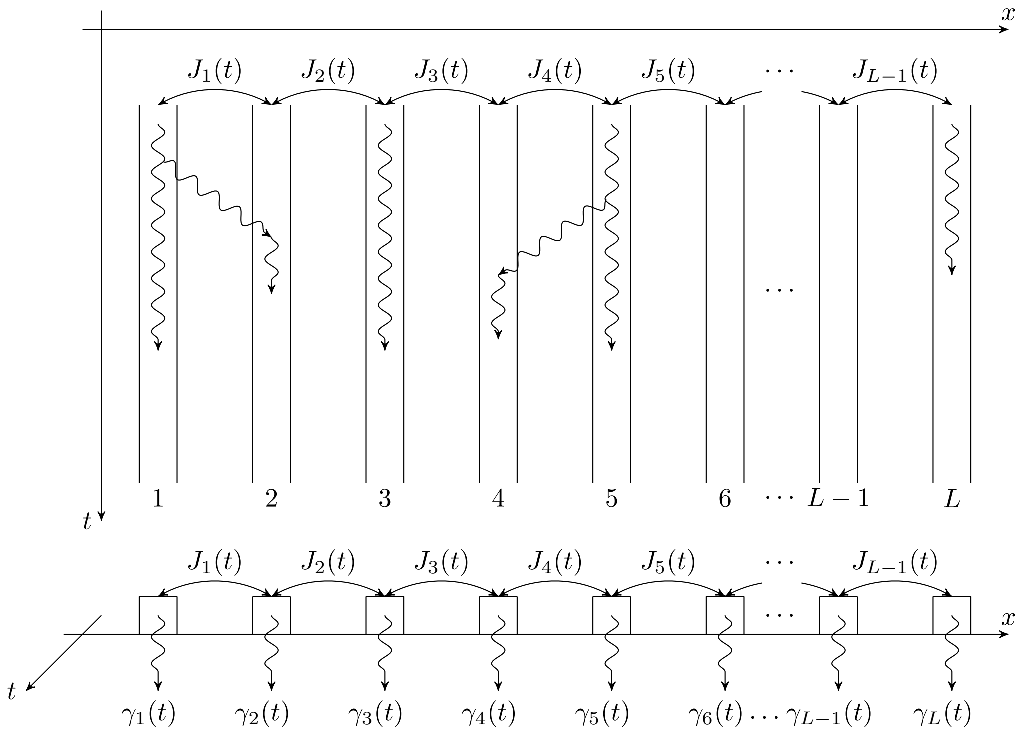

Figure 3.1: Waveguide model: Top view above and front view below: The Photons propagate through the waveguides and can tunnel to the neighbouring sites (top view). Besides, they can dissipate from the system (front view). Every waveguide is numbered from 1 to L. $J_i(t)$ denotes the hopping amplitude to the nearest neighbour sites and $\gamma_i(t)$ the dissipation strength.

Consequently, the most general form of the corresponding Hamiltonian we are going to consider is given by

$$H = -\sum_{i=1}^{L} J_i(t)(b_i^\dagger b_{i+1} + b_{i+1}^\dagger b_i)$$

with the hopping amplitude $J(t)$ and the bosonic creation and annihilation operators $b_i^\dagger$ and b_i for each site i[1]. These operators fulfil the following well known commutation relations:

$$[b_i, b_j] = 0 = [b_i^\dagger, b_j^\dagger] \ , \quad [b_i, b_j^\dagger] = \delta_{ij}$$

[1]Note that site $L+1$ is identified with site 1.

The Hilbert space $\mathcal{H}$ is uniquely defined by the vacuum $|0\rangle$ satisfying

$$b_i |0\rangle = 0 \quad \text{and} \quad \langle 0|0\rangle = 1$$

and n-particle states

$$b_{i_1}^\dagger \dots b_{i_n}^\dagger |0\rangle .$$

The particle number operator for site i is defined by

$$n_i := b_i^\dagger b_i.$$

In general, every waveguide has the property of losing photons. We are going to describe this in two different ways. On the one hand, we will use the Lindblad master equation as mentioned above. Then the dissipator $\mathcal{D}(\rho(t))$ takes the form

$$\mathcal{D}(\rho(t)) = \sum_{i=1}^{L} \gamma_i(t) \cdot (b_i \rho(t) b_i^\dagger - \frac{1}{2} b_i^\dagger b_i \rho(t) - \frac{1}{2} \rho(t) b_i^\dagger b_i)$$

where $\gamma_i(t)$ describes the strength of the dissipation at site i. On the other hand, we will add a non-hermitian part to the Hamiltonian, which causes a non-unitary time-evolution. As a consequence, the system is also losing photons so the dissipation can be described as well with this formalism. The corresponding Hamiltonian is then given by

$$H = -\sum_{i=1}^{L} J_i(t)(b_i^\dagger b_{i+1} + b_{i+1}^\dagger b_i) - i\frac{\gamma_i(t)}{2} b_i^\dagger b_i. \tag{3.1}$$

For the following calculations, we will always work in units such that $\hbar = 1$.

4 The homogeneous system

We start by considering the waveguides introduced in chapter 3 as a homogeneous system. Hence, the hopping amplitude $J(t)$ and the strength of the dissipation $\gamma(t)$ are the same for each waveguide. Note that $J_L(t) \neq 0$ and so we are working with periodic boundary conditions. We will solve the problem within the Lindblad formalism and also by using a non-hermitian Hamiltonian.

4.1 Solution within the Lindblad formalism

Solving the Lindblad equation in the Schrödinger picture for arbitrary numbers of waveguides and particles requires the diagonalization of matrices with varying dimensions. But in the Heisenberg picture we can get the information we are interested in also by solving a closed system of differential equations for the operators $b_i^\dagger(t)b_j(t)$. For these equations, we can find a general solution.

Plugging in an arbitrary operator $b_i^\dagger(t)b_j(t)$ in equation 2.2 and using the commutation relations we get

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}(b_i^\dagger(t)b_j(t)) =& -iJ_{i-1}(t)b_{i-1}^\dagger(t)b_j(t) + iJ_j(t)b_i^\dagger(t)b_{j+1}(t) - iJ_i(t)b_{i+1}^\dagger(t)b_j(t) \\ &+ iJ_{j-1}(t)b_i^\dagger(t)b_{j-1}(t) - \frac{1}{2}b_i^\dagger(t)b_j(t)(\gamma_i(t)+\gamma_j(t)).\end{aligned} \tag{4.1}$$

You can find an explicit derivation in the appendix A.1. For the homogeneous system this simplifies to

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}(b_i^\dagger(t)b_j(t)) =& -iJ(t)\left(b_{i-1}^\dagger(t)b_j(t) - b_i^\dagger(t)b_{j+1}(t) + b_{i+1}^\dagger(t)b_j(t) - b_i^\dagger(t)b_{j-1}(t)\right) \\ &- \gamma(t)b_i^\dagger(t)b_j(t).\end{aligned} \tag{4.2}$$

We solve this equation with a Fourier transformation. Hence, we define our new operators by

$$d_l := \frac{1}{\sqrt{L}}\sum_{j=1}^{L} e^{-2\pi ijl/L}b_j\ , \quad d_l^\dagger := \frac{1}{\sqrt{L}}\sum_{j=1}^{L} e^{2\pi ijl/L}b_j^\dagger. \tag{4.3}$$

These new operators still fulfil the same commutation relations as the original operators:

$$[d_a, d_b] = \frac{1}{L}\sum_{j,j'=1}^{L} e^{-2\pi ija/L}e^{-2\pi ij'b/L}\underbrace{[b_j, b_{j'}]}_{0} = 0$$

$$[d_a^\dagger, d_b^\dagger] = \frac{1}{L}\sum_{j,j'=1}^{L} e^{2\pi ija/L}e^{2\pi ij'b/L}\underbrace{[b_j^\dagger, b_{j'}^\dagger]}_{0} = 0$$

$$[d_a, d_b^\dagger] = \frac{1}{L}\sum_{j,j'=1}^{L} e^{-2\pi ija/L}e^{2\pi ij'b/L}\underbrace{[b_j, b_{j'}]}_{\delta j,j'} = \frac{1}{L}\sum_{j=1}^{L} e^{2\pi ij(b-a)/L} = \delta_{a,b}$$

For these new operators we obtain

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}(d_a^\dagger(t)d_b(t)) =& \frac{1}{L}\sum_{l,l'=1}^{L} e^{2\pi ila/L}e^{-2\pi il'b/L}\frac{\mathrm{d}}{\mathrm{d}t}(b_l^\dagger(t)b_{l'}(t)) \\
=& \frac{1}{L}\sum_{l,l'=1}^{L} e^{2\pi i(la-l'b)/L}\Big(-iJ(t)\Big(b_{l-1}^\dagger(t)b_{l'}(t) - b_l^\dagger(t)b_{l'+1}(t) + b_{l+1}^\dagger(t)b_{l'}(t) \\
& - b_l^\dagger(t)b_{l'-1}(t)\Big) - \gamma b_l^\dagger(t)b_{l'}(t)\Big) \\
=& \Big(-iJ(t)(e^{2\pi ia/l} - e^{2\pi ib/l} + e^{-2\pi ia/l} - e^{-2\pi ib/l}) - \gamma(t)\Big)\, d_a^\dagger(t)d_b(t) \\
=& \left(-2iJ(t)\left(\cos\left(2\pi\frac{a}{L}\right) - \cos\left(2\pi\frac{b}{L}\right)\right) - \gamma(t)\right) d_a^\dagger(t)d_b(t)
\end{aligned}
$$

with the unique solution

$$
d_a^\dagger(t)d_b(t) = \exp\left\{\int_0^t -2iJ(t')\left(\cos\left(2\pi\frac{a}{L}\right) - \cos\left(2\pi\frac{b}{L}\right)\right) - \gamma(t')\mathrm{d}t'\right\} d_a^\dagger(0)d_b(0).
$$

For the expectation value of the original operators we thus have

$$
\begin{aligned}
\langle b_l^\dagger(t)b_{l'}(t)\rangle =& \frac{1}{L}\sum_{a,b=1}^{L}\langle d_a^\dagger(0)d_b(0)\rangle\cdot\exp\{-2\pi ila/L\}\cdot\exp\{2\pi il'b/L\} \\
& \cdot\exp\left\{\int_0^t -2iJ(t')\left(\cos\left(2\pi\frac{a}{L}\right) - \cos\left(2\pi\frac{b}{L}\right)\right) - \gamma(t')\mathrm{d}t'\right\}
\end{aligned}
$$

where the constant $\langle d_a^\dagger(0)d_b(0)\rangle$ only depends on the initial conditions in form of the density matrix and relates to the expectation value of the original operators by

$$
\langle d_a^\dagger(0)d_b(0)\rangle = \frac{1}{L}\sum_{l,l'=1}^{L} e^{2\pi ila/L}e^{-2\pi il'b/L}\langle b_l^\dagger(0)b_{l'}(0)\rangle.
$$

One interesting quantity is the decay of the total number of particles in the system, which can be evaluated by

$$
\begin{aligned}
\sum_{i=1}^{L}\langle n_i(t)\rangle =& \sum_{l=1}^{L}\langle b_l^\dagger(t)b_l(t)\rangle \\
=& \sum_{a,b=1}^{L}\underbrace{\frac{1}{L}\sum_{l=1}^{L}\exp\{-2\pi il(a-b)/L\}}_{\delta_{a,b}}\cdot\langle d_a^\dagger(0)d_b(0)\rangle \\
& \cdot\exp\left\{\int_0^t -2iJ(t')\left(\cos\left(2\pi\frac{a}{L}\right) - \cos\left(2\pi\frac{b}{L}\right)\right) - \gamma(t')\mathrm{d}t'\right\} \\
=& \sum_{a=1}^{L}\langle d_a^\dagger(0)d_a(0)\rangle\cdot\exp\left\{\int_0^t -2iJ(t')\left(\cos\left(2\pi\frac{a}{L}\right) - \cos\left(2\pi\frac{a}{L}\right)\right) - \gamma(t')\mathrm{d}t'\right\} \\
=& \sum_{l,l'=1}^{L}\underbrace{\frac{1}{L}\sum_{a}^{l}\exp\{2\pi ia(l-l')/L\}}_{\delta_{l,l'}}\cdot\langle b_l^\dagger(0)b_{l'}(0)\rangle\cdot\exp\left\{-\int_0^t\gamma(t')\mathrm{d}t'\right\} \\
=& \sum_{l=1}^{L}\langle b_l^\dagger(0)b_l(0)\rangle\cdot\exp\left\{-\int_0^t\gamma(t')\mathrm{d}t'\right\}.
\end{aligned}
\tag{4.4}
$$

We conclude that for a time-independent dissipation the decay of the total number of particles goes exponentially with γ. You can find an example in figure 4.1.

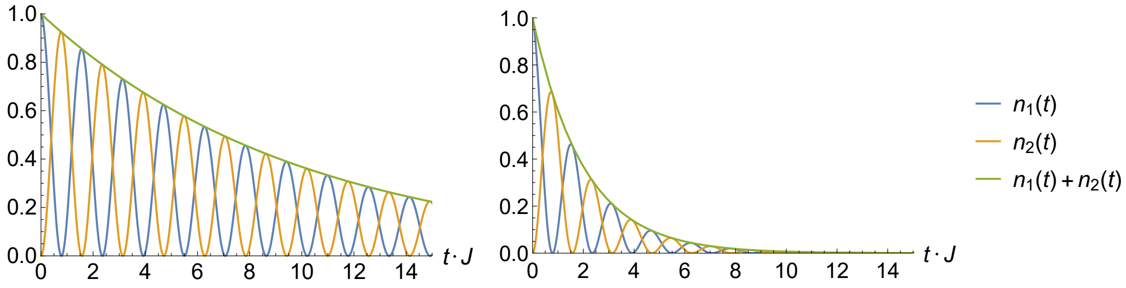

Figure 4.1: Evolution of the particle numbers in time by considering two waveguides with one particle starting on the first one. The analytic solutions are following from equation 4.4 in the Lindblad formalism: $n_1(t) = e^{-\gamma t}\cos^2(2Jt)$, $n_2(t) = e^{-\gamma t}\sin^2(2Jt)$ and $n_1(t) + n_2(t) = e^{-\gamma t}$. Parameters: left: $\gamma = 0.1 \cdot J$, right: $\gamma = 0.5 \cdot J$.

For a time-dependent dissipation like

$$\gamma(t) = \gamma_0 \cdot (1 + \sin(\omega t))$$

the decay takes the form

$$\sum_{i=1}^{L} \langle n_i(t) \rangle = \sum_{l=1}^{L} \langle b_l^\dagger(0) b_l(0) \rangle \cdot e^{-\gamma_0 t + \frac{\gamma_0 \cos(\omega t)}{\omega} - \frac{\gamma_0}{\omega}}.$$

An example for this behaviour is depicted in figure 4.2.

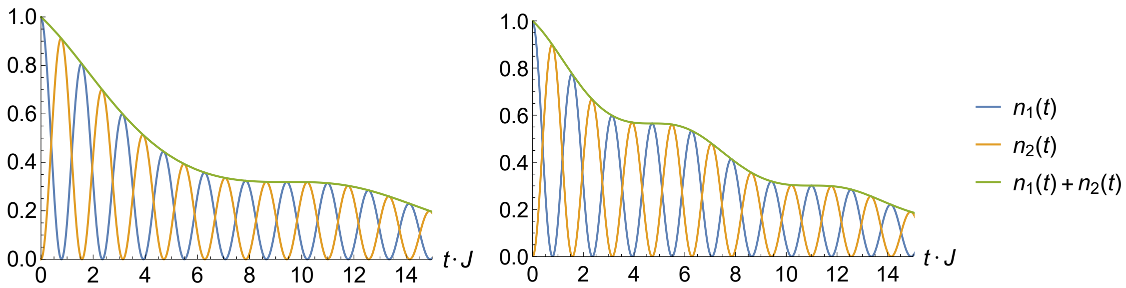

Figure 4.2: Evolution of the particle numbers in time by considering two waveguides with one particle starting on the first one with $\gamma_0 = 0.1 \cdot J$. The analytic solutions are following from equation 4.4 in the Lindblad formalism: $n_1(t) = e^{-\gamma_0 t + \frac{\gamma_0 \cos(\omega t)}{\omega} - \frac{\gamma_0}{\omega}} \cos^2(2Jt)$, $n_2(t) = e^{-\gamma_0 t + \frac{\gamma_0 \cos(\omega t)}{\omega} - \frac{\gamma_0}{\omega}} \sin^2(2Jt)$ and $n_1(t)+n_2(t) = e^{-\gamma_0 t + \frac{\gamma_0 \cos(\omega t)}{\omega} - \frac{\gamma_0}{\omega}}$. Parameters: left: $\omega = 0.5 \cdot J$, right: $\omega = 1 \cdot J$.

As one would expect, the system exponentially loses particles in the time-independent case. If one modulates the dissipation and makes it, for example, periodic in time the loss of the particles changes accordingly.

4.2 Solution with non-hermitian Hamiltonian

For the diagonalization of the non-hermitian Hamiltonian 3.1 we will use the same Fourier transformation for our operators as defined in equation 4.3. One then gets the diagonalized Hamiltonian

$$H = \underbrace{\left(-2J(t)\cos\left(2\pi\frac{l}{L}\right) - i\frac{\gamma(t)}{2}\right)}_{E_l(t)} d_l^\dagger d_l.$$

For the expectation value of our operators, this leads to

$$\begin{aligned}\langle b_a^\dagger(t) b_b(t)\rangle =& \frac{1}{L^2}\sum_{l,l',k,k'=1}^{L} \exp\{2\pi i l/L(b-k)\}\cdot\exp\{-2\pi i l'/L(a-k')\}\cdot\exp\left\{i\int_0^t E_{l'}(t')^* \mathrm{d}t'\right\} \\ &\cdot\exp\left\{-i\int_0^t E_l(t')\mathrm{d}t'\right\}\langle b_{k'}^\dagger(0) b_k(0)\rangle\end{aligned}$$

where $\langle b_{k'}^\dagger(0) b_k(0)\rangle$ depends on the initial conditions, which are now given in form of an initial state. For the decay of the total number of particles in the system, one obtains

$$\sum_{i=1}^{L}\langle n_i(t)\rangle = \sum_{k=1}^{L}\langle b_k^\dagger(0) b_k(0)\rangle\cdot\exp\left\{-\int_0^t \gamma(t')\mathrm{d}t'\right\},$$

which is equivalent to the solution one gets out of the Lindblad formalism.

5 Systems with dissipative defects

As an example for a more interesting system, we will have a look at two waveguides but only one of them having the property of losing photons. The hopping amplitude J is assumed to be constant. You can find a graphical illustration in figure 5.1.

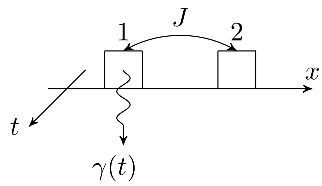

Figure 5.1: Front view of our two waveguide model. Photons can only dissipate from the first waveguide. The hopping amplitude J is constant.

Again, we will assume periodic boundary conditions. In this case, the Lindblad equation takes the form

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho(t) = -2iJ[(b_1^\dagger b_2 + b_2^\dagger b_1), \rho(t)] + \gamma(t) \cdot (b_1 \rho(t) b_1^\dagger - \frac{1}{2} b_1^\dagger b_1 \rho(t) - \frac{1}{2}\rho(t) b_1^\dagger b_1).$$

We want to find a general solution for the density matrix restricted to one-particle states and the vacuum. As a basis for this subspace of the complete Hilbert space, we choose the states

$$|0,0\rangle := |0\rangle \ , \quad |1,0\rangle := b_1^\dagger |0\rangle \ , \quad |0,1\rangle := b_2^\dagger |0\rangle .$$

The only non-vanishing part of the density matrix then reads

$$\rho = \begin{array}{c} \\ |1,0\rangle \\ |0,1\rangle \\ |0,0\rangle \end{array} \begin{array}{c} \begin{array}{ccc} \langle 1,0| & \langle 0,1| & \langle 0,0| \end{array} \\ \begin{pmatrix} a & b & e \\ c & d & f \\ g & h & i \end{pmatrix} \end{array}.$$

Note that our Lindblad equation really reduces to an equation in this three-dimensional subspace.

5.1 Time-independent dissipative defect

First of all, we consider a time-independent dissipation strength γ. Again, we solve the problem within the Lindblad formalism and also by using a non-hermitian Hamiltonian.

5.1.1 Solution within the Lindblad formalism

We can rewrite the Lindblad equation in the form

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho(t) = M \cdot \rho(t)$$

where ρ now represents a nine-dimensional vector containing all the components of the density matrix. With the matrix exponential the unique solution is given by

$$\rho(t) = e^{Mt}\rho(0).$$

Diagonalizing M this can be written as

$$\rho(t) = \sum_n \alpha_n e^{\lambda_n t} \rho_n \quad \text{with} \quad \rho(0) = \sum_n \alpha_n \cdot \rho_n$$

where ρ_n denotes the eigenvectors and λ_n the corresponding eigenvalues. For our problem, the Lindblad equation explicitly looks like this:

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \\ i \end{pmatrix} = \underbrace{\begin{pmatrix} -\gamma & -2iJ & 2iJ & 0 & 0 & 0 & 0 & 0 & 0 \\ -2iJ & -\gamma/2 & 0 & 2iJ & 0 & 0 & 0 & 0 & 0 \\ 2iJ & 0 & -\gamma/2 & -2iJ & 0 & 0 & 0 & 0 & 0 \\ 0 & 2iJ & -2iJ & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\gamma/2 & 2iJ & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2iJ & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\gamma/2 & -2iJ & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -2iJ & 0 & 0 \\ \gamma & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}}_{M} \cdot \begin{pmatrix} a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \\ i \end{pmatrix}$$

Diagonalization of the matrix M leads to the following eigenvalues:

$$0,\ -\frac{\gamma}{2},\ -\frac{\gamma}{2},\ \frac{1}{4}\left(-\gamma - \sqrt{\gamma^2 - 64J^2}\right),\ \frac{1}{4}\left(-\gamma - \sqrt{\gamma^2 - 64J^2}\right)$$
$$\frac{1}{4}\left(-\gamma + \sqrt{\gamma^2 - 64J^2}\right),\ \frac{1}{2}\left(-\gamma - \sqrt{\gamma^2 - 64J^2}\right),\ \frac{1}{4}\left(-\gamma + \sqrt{\gamma^2 - 64J^2}\right),\ \frac{1}{2}\left(-\gamma + \sqrt{\gamma^2 - 64J^2}\right)$$

The eigenvectors can be found in the appendix A.2.

Now we can have a closer look on the solution for a certain initial condition. We consider the initial condition that the particle starts at the second waveguide. In this case, the density matrix is given by [1]

$$\rho(0) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{5.1}$$

Consequently, the particle is not effected by the dissipation at the beginning. It is now interesting to see how the particle is going to evolve in the two waveguides. Therefore, we will study the expectation value of the particle number operators on site one and site two. One gets the following equations:

$$\langle n_1(t)\rangle = \frac{64J^2 \text{Sinh}\left(\frac{1}{4}\sqrt{\gamma^2 - 64J^2}t\right)^2}{\gamma^2 - 64J^2} e^{\frac{-\gamma t}{2}} \tag{5.2}$$
$$\langle n_2(t)\rangle = \frac{\left(\gamma^2 - 32J^2\right)\text{Cosh}\left(\frac{1}{2}\sqrt{\gamma^2 - 64J^2}t\right) + \gamma\sqrt{\gamma^2 - 64J^2}\text{Sinh}\left(\frac{1}{2}\sqrt{\gamma^2 - 64J^2}t\right) - 32J^2}{\gamma^2 - 64J^2} e^{\frac{-\gamma t}{2}}$$

For describing the features of the decay, we distinguish the case $\gamma < 8J$ where the eigenvalues have an imaginary part from the case $\gamma \geq 8J$. For $\gamma < 8J$ one can see that there is again an overall exponential decay of the total particle number, this time together with small oscillations, which can be seen in figure 5.2. The parameters are the same as in the previous example for the homogeneous system depicted in figure 4.1. Note that the exponential decay goes with $\gamma/2$ due to the fact that the dissipation is only present on one of the two sites. Up to the factor of two in the decay, it qualitatively agrees with the one of the homogeneous system.

[1] One can easily check that this is a valid density matrix as discussed in chapter 2.

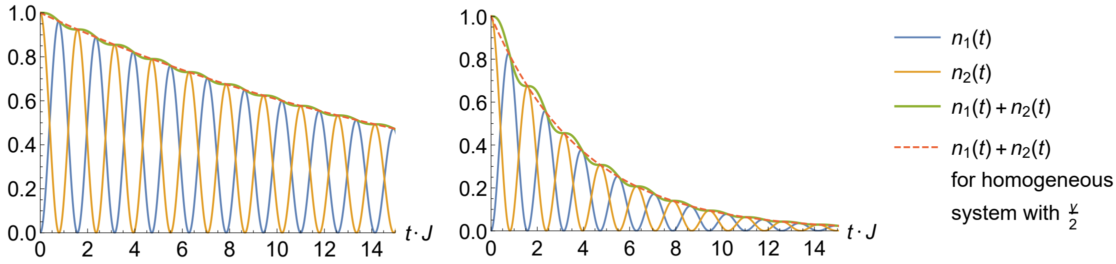

Figure 5.2: Evolution of the particle numbers in time for the two waveguide system depicted in figure 5.1 with one particle starting on the second non-dissipative site. The analytic solutions of the Lindblad equation are given in equation 5.2 and for the associated homogeneous one they follow from equation 4.4. Parameters: left: $\gamma = 0.1 \cdot J$, right: $\gamma = 0.5 \cdot J$.

For $\gamma \geq 8J$ we observe a different behaviour of the system for different dissipation strengths in contrast to the homogeneous system. By increasing γ the strength of the exponential decay decreases. In order to understand this, we look at the structure of the contributing eigenvalues which, for our inital condition, are

$$0\,, \quad -\gamma/2\,, \quad \frac{1}{2}\left(-\gamma - \sqrt{\gamma^2 - 64J^2}\right) \quad \text{and} \quad \frac{1}{2}\left(-\gamma + \sqrt{\gamma^2 - 64J^2}\right). \tag{5.3}$$

Note that the first eigenvalue does not lead to a decay and corresponds to the pure vacuum state (see the corresponding eigenvector in the appendix A.2). The other eigenvalues are responsible for the decay. The asymptotic decay for large t is governed by the eigenvalue with the largest real part. One can see that exactly at $\gamma = 8J$ there is a breaking point from which on the last eigenvalue determines the decay. This can be seen in figure 5.3.

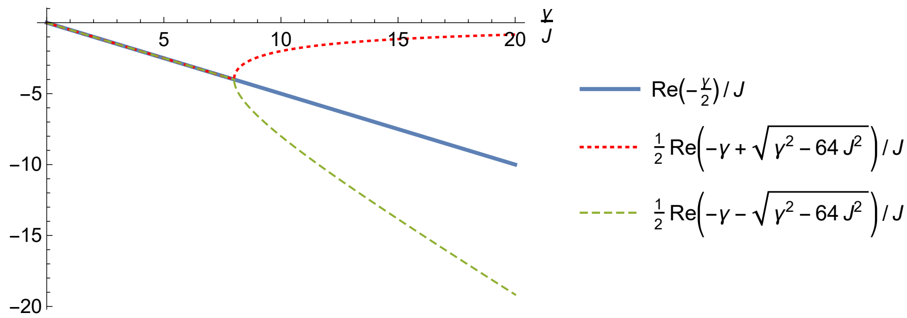

Figure 5.3: Real part of the contributing eigenvalues given in 5.3 in the Lindblad formalism.

For large γ one finds that the decay approximately goes with

$$\begin{aligned}\frac{1}{2}\left(-\gamma + \sqrt{\gamma^2 - 64J^2}\right) &= \frac{1}{2}\left(-\gamma + \gamma\sqrt{1 - \left(\frac{8J}{\gamma}\right)^2}\right)\\ &= \frac{1}{2}\left(-\gamma + \gamma\left(1 - \frac{1}{2}\left(\frac{8J}{\gamma}\right)^2\right)\right) + \mathcal{O}\left(\frac{1}{\gamma^3}\right)\\ &= -\frac{16J^2}{\gamma} + \mathcal{O}\left(\frac{1}{\gamma^3}\right).\end{aligned} \tag{5.4}$$

The effect that the decay goes with $1/\gamma$ in the large γ limit is called the quantum Zeno effect. Following Misra and Sudarshan (1977) and Fröml (2019), it describes that transitions between quantum states are suppressed by a frequent measurement of the quantum system. The measurement process corresponds in this case to the loss of photons.

Figure 5.4 depicts the quality of our approximation in the large γ limit. In the limit $\gamma \to \infty$ the expectation value of the number of particles on site two goes to one while site one stays unpopulated. Hence, the hopping is completely suppressed for finite times t.

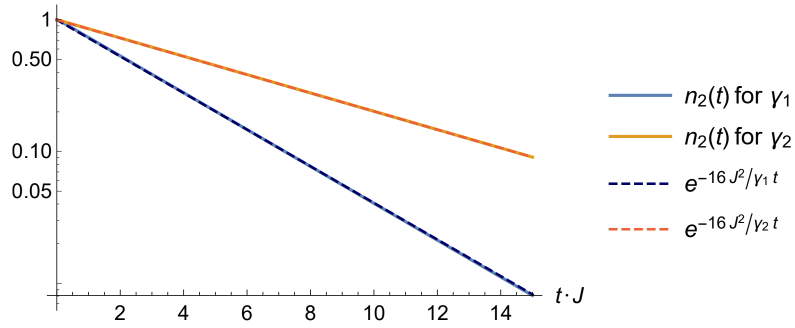

Figure 5.4: The exponential decay of the particle number of the second non-dissipative waveguide for $\gamma_1 = 50 \cdot J$ and $\gamma_2 = 100 \cdot J$ following from equation 5.2 in the Lindblad formalism in a logarithmic plot. In comparison one can see the approximated decay in the large γ limit (equation 5.4).

A comment regarding the importance of the initial conditions for the occurance of the Zeno effect is made at the end of the next paragraph.

5.1.2 Solution with non-hermitian Hamiltonian

Again, we now switch the formalism and describe this system with a non-hermitian Hamiltonian, which is given by

$$H = -2J(b_1^\dagger b_2 + b_2^\dagger b_1) - i\frac{\gamma}{2} b_1^\dagger b_1.$$

Restricting to the one-particle Hilbert space this can be written in an explicit matrix form

$$H = \begin{pmatrix} -i\frac{\gamma}{2} & -2J \\ -2J & 0 \end{pmatrix}.$$

Diagonalization of the Hamiltonian gives us the following eigenvectors and eigenvalues:

$$\begin{pmatrix} -\frac{-i\gamma - \sqrt{-\gamma^2+64J^2}}{8J} \\ 1 \end{pmatrix}, \begin{pmatrix} -\frac{-i\gamma + \sqrt{-\gamma^2+64J^2}}{8J} \\ 1 \end{pmatrix}$$

$$\frac{1}{4}\left(-i\gamma - \sqrt{-\gamma^2 + 64J^2}\right), \ \frac{1}{4}\left(-i\gamma + \sqrt{-\gamma^2 + 64J^2}\right) \tag{5.5}$$

The imaginary part of the eigenvalues is shown in figure 5.5. Again, we find that the breaking point is at $\gamma = 8J$.

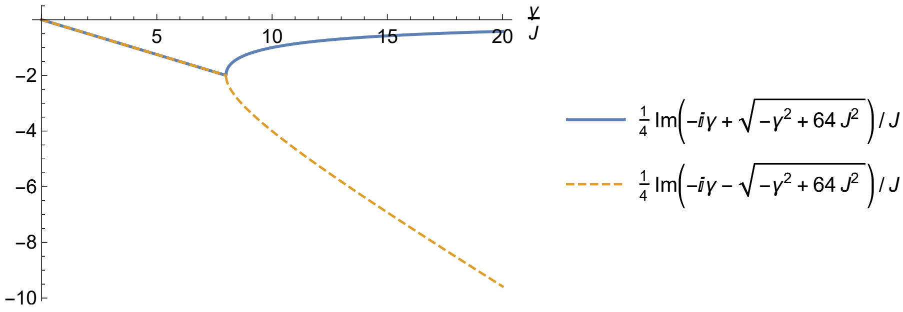

Figure 5.5: Imaginary part of the eigenvalues given in 5.5 of the non-hermitian Hamiltonian.

In fact, we observe the same behaviour of the system for small dissipation strengths as well as for high dissipation strengths in comparison to the solution of the Lindblad equation. Therefore, our approximation for the slope in the large γ limit is also valid in this case. An example for that is shown in figure 5.6.

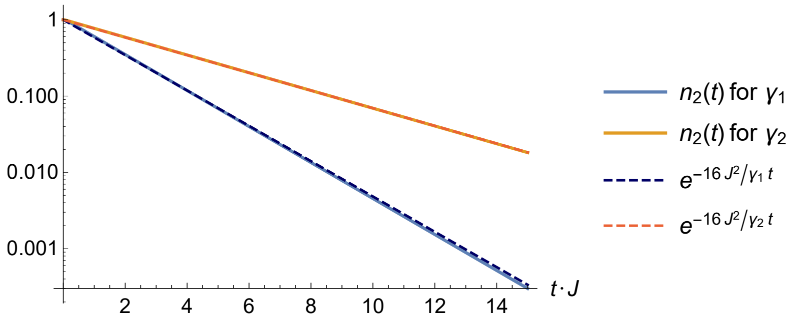

Figure 5.6: The exponential decay of the particle number of the second non-dissipative waveguide for $\gamma_1 = 30 \cdot J$ and $\gamma_2 = 60 \cdot J$ considering the non-hermitian Hamiltonian in a logarithmic plot. In comparison one can see the approximated decay in the large γ limit (equation 5.4).

Note that in both cases we considered the initial condition that the particle starts on the non-dissipative site. That is actually quite important in order to obtain the Zeno effect. If the particle would start at the dissipative site it would just get immediately lost for high dissipation strengths.

In the next step, we are going to change the dissipation by making it periodic in time.

5.2 Time-dependent dissipative defect

Now we want to consider a time-depedent dissipation. We stick to the previous two waveguide system depicted in figure 5.1 but modify the dissipation on the first waveguide to

$$\gamma(t) = \gamma_0 \cdot (1 + \sin(\omega t)).$$

We will first solve the resulting Lindblad equation numerically. After that we use Floquet theory to solve the problem described by the non-hermitian Hamiltonian.

5.2.1 Numerical solution within the Lindblad formalism

We solve the set of differential equations in the Heisenberg picture following from equation 4.1 numerically with MATHEMATICA. The system of differential equations is explicitly given by

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\langle n_1(t)\rangle + \gamma(t)\langle n_1(t)\rangle &= -2iJ(\langle b_2^\dagger(t)b_1(t)\rangle - \langle b_1^\dagger(t)b_2(t)\rangle),\\
\frac{\mathrm{d}}{\mathrm{d}t}\langle b_1^\dagger(t)b_2(t)\rangle + \frac{\gamma(t)}{2}\langle b_1^\dagger(t)b_2(t)\rangle &= -2iJ(\langle n_2(t)\rangle - \langle n_1(t)\rangle),\\
\frac{\mathrm{d}}{\mathrm{d}t}\langle b_2^\dagger(t)b_1(t)\rangle + \frac{\gamma(t)}{2}\langle b_2^\dagger(t)b_1(t)\rangle &= -2iJ(\langle n_1(t)\rangle - \langle n_2(t)\rangle),\\
\frac{\mathrm{d}}{\mathrm{d}t}\langle n_2(t)\rangle &= -2iJ(\langle b_1^\dagger(t)b_2(t)\rangle - \langle b_2^\dagger(t)b_1(t)\rangle).
\end{aligned}$$

As initial condition, we, again, take the density matrix 5.1, which corresponds to one particle being on the second non-dissipative waveguide. This gives us the matrix components

$$\begin{aligned}
\langle n_1(0)\rangle &= 0,\\
\langle b_1^\dagger(0)b_2(0)\rangle &= 0,\\
\langle b_2^\dagger(0)b_1(0)\rangle &= 0,\\
\langle n_2(0)\rangle &= 1.
\end{aligned}$$

First, we have a look at the solution for small values of the dissipation strength. Again, we obtain a difference of a factor of two in the exponential decay and some oscillations in comparison to the solution of the corresponding homogeneous system. We can find an example for that in figure 5.7. This one considers the same parameters as the example of the homogeneous system depicted in figure 4.2.

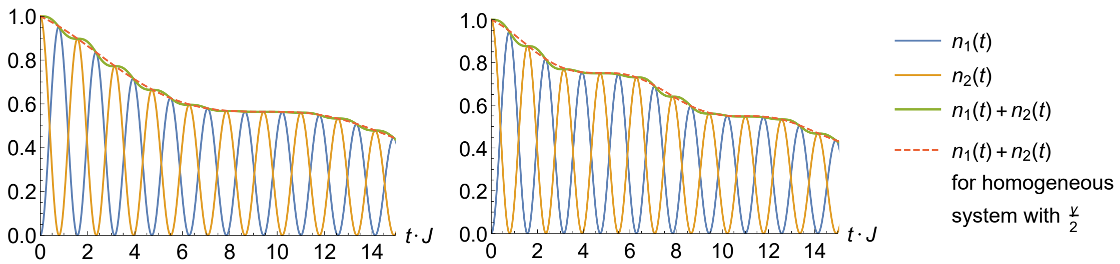

Figure 5.7: Evolution of the particle numbers in time for the two waveguide system depicted in figure 5.1 with one particle starting on the second non-dissipative site. The dissipation on the first waveguide is periodic in time with $\gamma_0 = 0.1 \cdot J$. The expectation values of $n_1(t)$ and $n_2(t)$ follow from the numerical solution of the corresponding Lindblad equation in the Heisenberg picture. The analytic solutions of the Lindblad equation for the associated homogeneous system follow from equation 4.4. Parameters: left: $\omega = 0.5 \cdot J$, right: $\omega = 1 \cdot J$.

Secondly, we discover for high dissipation strengths several resonances at specific values of the frequency ω where the strength of the decay strongly increases. This effect can be seen in figure 5.8.

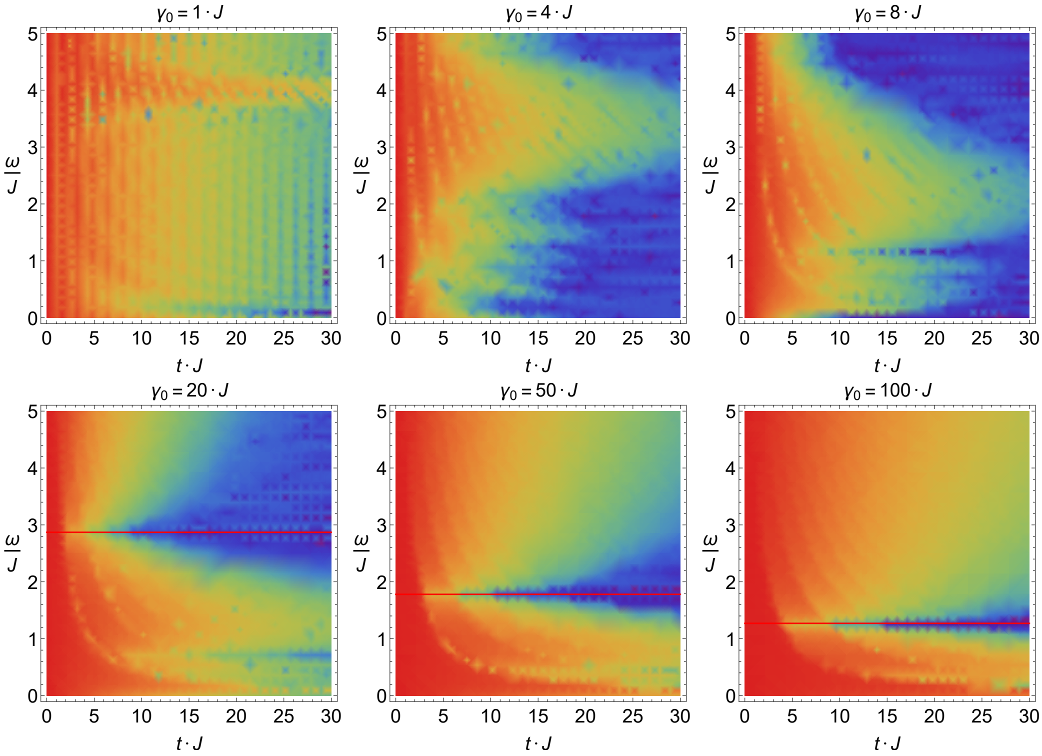

Figure 5.8: Evolution of the particle number on the second non-dissipative waveguide in time as a function of the frequency ω of the dissipation strength $\gamma(t)$ for different γ_0. These numerical solutions follow from the Lindblad equation in the Heisenberg picture. The red colour indicates a high number of particles whereas the blue colour corresponds to a low number of particles. The red lines in the second row clarify the frequency where the resonance occurs: $\omega \approx 2.87$, $\omega \approx 1.78$ and $\omega \approx 1.27$ from left to right. Note that $n_2(t)$ is plotted logarithmically.

Exemplary, we will now have a closer look on the behaviour of the system for $\gamma_0 = 50 \cdot J$. In figure 5.8 you can find a sharp peak around $\omega = 1.78 \cdot J$. Hence, we consider the expectation value of $n_1(t)$ and $n_2(t)$ for this frequency in detail and in comparison also for two frequencies around the resonance. Figure 5.9 shows that in general the expectation value of $n_2(t)$ starts decreasing rapidly for $\sin(\omega t)$ getting negative. In this cases, the expectation value of $n_1(t)$ peaks around the minimum of $\sin(\omega t)$. In the system where $\omega = 1.78 \cdot J$ the particle is completely lost after one period $2\pi/\omega$. With other frequencies around the resonance the particle stays longer in the system. Qualitatively, this again corresponds to the Zeno effect. At the beginning, the system is already in the Zeno limit because of the high dissipation strength. As a consequence the hopping of the particle to the first waveguide is suppressed and $n_1(t)$ is zero. But when the $\sin(\omega t)$ gets negative the whole dissipation strength becomes small. At this point the system is no longer in the Zeno limit and the hopping of the particle to the first waveguide is more likely then before. Then the particle gets lost very quickly.

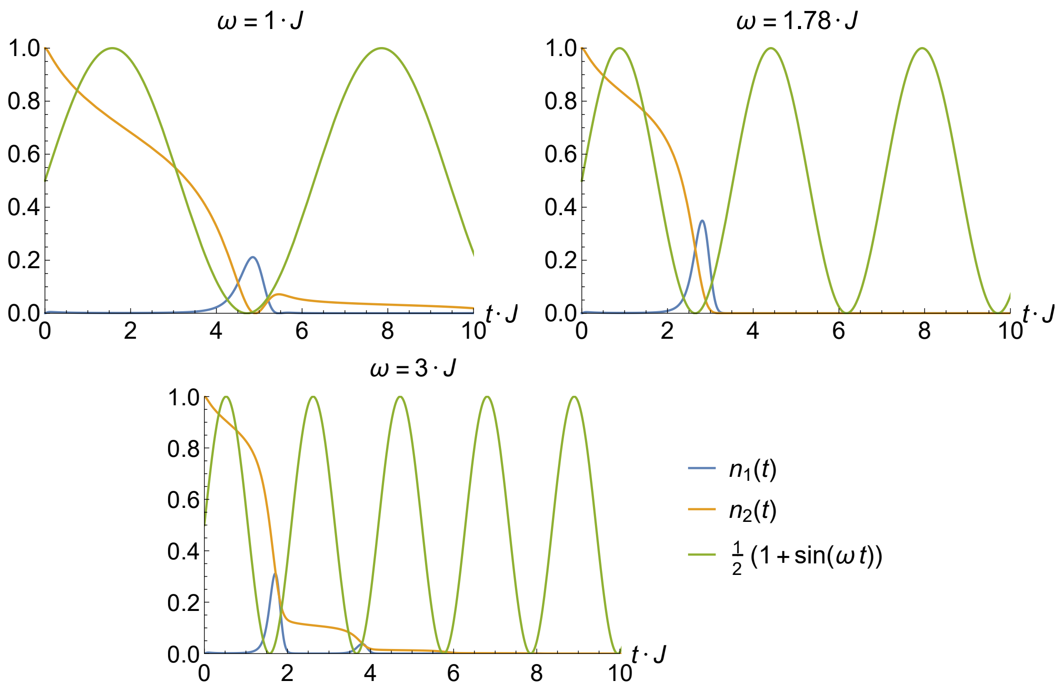

Figure 5.9: Evolution of the particle number on the first dissipative and second non-dissipative waveguide for $\gamma_0 = 50 \cdot J$ in time for different frequencies ω. The considered expectation values of $n_1(t)$ and $n_2(t)$ follow from the numerical solution of the Lindblad equation.

5.2.2 Numerical solution within Floquet theory

In order to get an approximate analytic solution, we now try to solve the problem for the non-hermitian Hamiltonian

$$H(t) = -2J(b_1^\dagger b_2 + b_2^\dagger b_1) - i\frac{\gamma(t)}{2} b_1^\dagger b_1.$$

We will, again, restrict to the two dimensional one-particle Hilbert space. For the choosen time periodic dissipation $\gamma(t) = \gamma_0 \cdot (1 + \sin(\omega t))$ the Hamiltonian is also periodic in time

$$H(t + 2\pi/\omega) = H(t),$$

and thus, we try to solve the problem by using Floquet theory. One can find a good introduction to Floquet theory in general in Holthaus (2015). For our purpose, we mainly follow Cherpakova (2018). In analogy to Bloch's theorem, a basis of solutions for the Schrödinger equation is given by

$$|\Psi_\alpha(t)\rangle = e^{-i\varepsilon_\alpha t} |u_\alpha(t)\rangle$$

with the Floquet states $|u_\alpha(t)\rangle$ also having period $2\pi/\omega$. The ϵ_α are called quasienergies. Note that the quasienergies defined by this relation are only unique (given a state $|\psi_\alpha(t)\rangle$) up to multiples of ω.

The Floquet modes, thus, need to satisfy

$$\left(H(t) - i\frac{\partial}{\partial t}\right) |u_\alpha(t)\rangle = \varepsilon_\alpha |u_\alpha(t)\rangle .$$

Since the Hamiltonian as well as the Floquet modes are time periodic, we can use a discrete Fourier transformation and write them as

$$H(t) = \sum_{k=-\infty}^{\infty} e^{-ik\omega t} H_k$$
$$|u_\alpha(t)\rangle = \sum_{k=-\infty}^{\infty} e^{-ik\omega t} \left|u_\alpha^k\right\rangle .$$

For the modes we get the time-independent Floquet equation

$$(H_0 - k\omega)\left|u_\alpha^k\right\rangle + \sum_{l\neq 0} H_l \left|u_\alpha^{k-l}\right\rangle = \varepsilon_\alpha \left|u_\alpha^k\right\rangle .$$

In our case, the decomposition of the Hamiltonian reads

$$H(t) = \underbrace{\begin{pmatrix} -i\frac{\gamma_0}{2} & -2J \\ -2J & 0 \end{pmatrix}}_{H_0} + \underbrace{\begin{pmatrix} \frac{\gamma_0}{4} & 0 \\ 0 & 0 \end{pmatrix}}_{H_1} e^{-i\omega t} + \underbrace{\begin{pmatrix} -\frac{\gamma_0}{4} & 0 \\ 0 & 0 \end{pmatrix}}_{H_{-1}} e^{i\omega t}$$

and so the equation for the modes is

$$\begin{pmatrix} \ddots & & & & \\ H_1 & H_0+\omega I & H_{-1} & & \\ & H_1 & H_0 & H_{-1} & \\ & & H_1 & H_0-\omega I & H_{-1} \\ & & & & \ddots \end{pmatrix} \begin{pmatrix} \vdots \\ u_\alpha^{-1} \\ u_\alpha^0 \\ u_\alpha^1 \\ \vdots \end{pmatrix} = \varepsilon_\alpha \begin{pmatrix} \vdots \\ u_\alpha^{-1} \\ u_\alpha^0 \\ u_\alpha^1 \\ \vdots \end{pmatrix} .$$

The idea now is to truncate this matrix to a $2(2n+1)$ dimensional matrix and solve the resulting finite dimensional eigenvalue problem. Considering enough frequency modes (n big enough) leads to a convergence of the eigenvalues in the sense that all eigenvalues can be generated by taking two eigenvalues and adding multiples of ω. We will always pick the two eigenvalues ϵ_1 and ϵ_2 from some Brillouin zone

$$\omega_0 - \omega/2 < \mathrm{Re}(\epsilon_\alpha) \leq \omega_0 + \omega/2.$$

With the two chosen eigenvalues and the corresponding eigenvectors, we have found an approximation for the two basis states $|\psi_\alpha(t)\rangle$, which solve the Schrödinger equation. A general solution can then be written as

$$|\Psi(t)\rangle = \sum_{\alpha=1}^{2}\sum_{k=-n}^{n} c_\alpha e^{-i\varepsilon_\alpha^k t} \left|u_\alpha^k\right\rangle ,$$

where the coefficients c_α are determined by the initial conditions.

The convergence is depending much on the considered values for the dissipation strength and the frequency. Depending on the parameters, we need to take at least ten to fifteen frequency modes into account. Figure 5.10 shows that, for smaller numbers of frequency modes n, you cannot determine significant results. Thus, the following results include fifteen frequency modes.

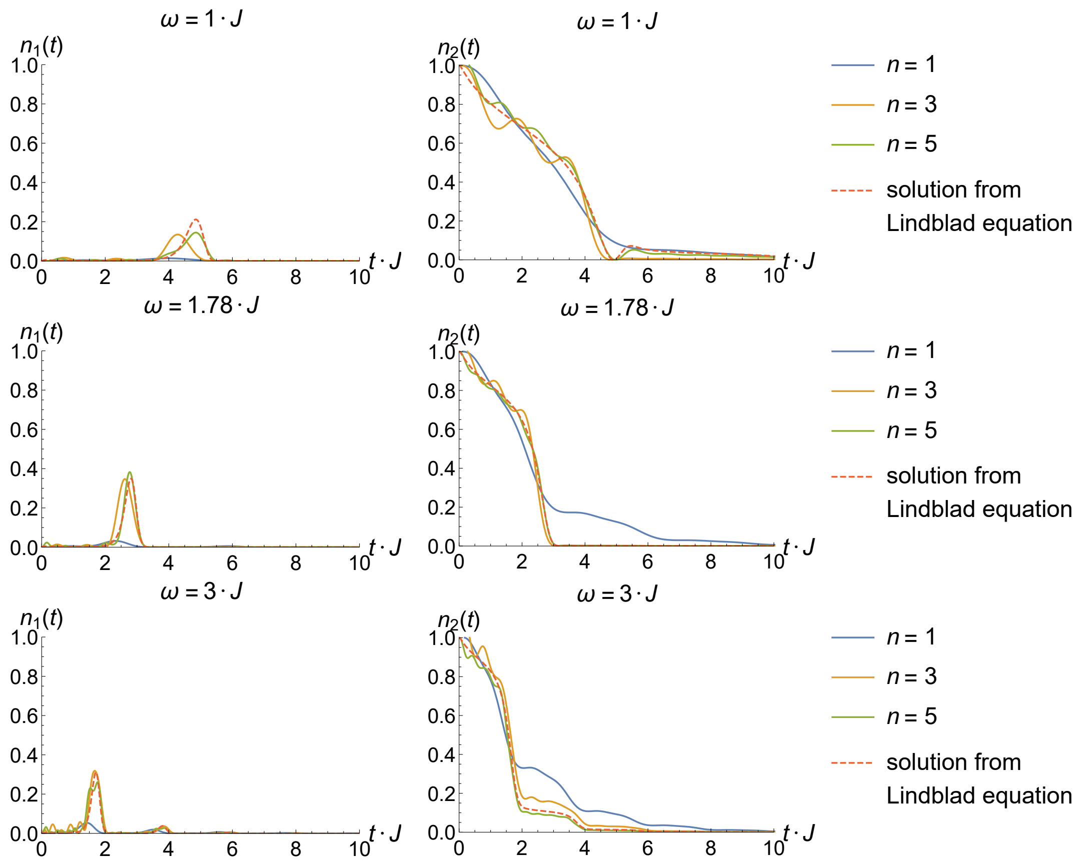

Figure 5.10: Evolution of the particle number on the first dissipative and second non-dissipative waveguide for $\gamma_0 = 50 \cdot J$ in time for different frequencies ω. The numerical solution of the corresponding Lindblad equation is compared to the one following by solving the problem with Floquet theory for different numbers of frequency modes n.

First of all, one again obtains the same behaviour of the system as before with the Lindblad formalism. You can find an example for that in figure 5.11. The expectation values from both formalisms lie on top of each other which verifies the convergence of the eigenvalues. To underline this, we used the same parameters as in the previous example in figure 5.10.

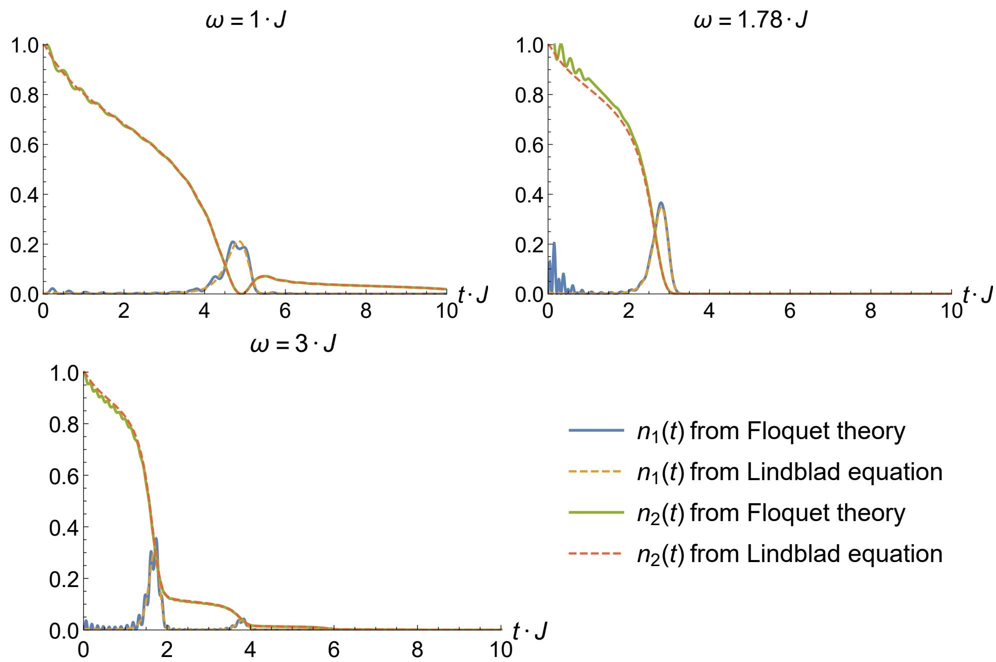

Figure 5.11: Evolution of the particle number on the first dissipative and second non-dissipative waveguide for $\gamma_0 = 50 \cdot J$ in time for different frequencies ω. The numerical solution of the corresponding Lindblad equation is compared to the one following by solving the problem with Floquet theory. In this case, fifteen frequency modes were taken into account.

Of interest are now the quasienergies of the Floquet-modes. In figure 5.12 one sees the imaginary part of the eigenvalue with the largest imaginary part depending on the frequency ω for the same fixed values of γ_0 as in figure 5.8. This value indicates the large time dynamics of the system and corresponds to the expected decay. There you can find the same resonance peaks for high values of γ_0 as in figure 5.8 represented by the red lines. Besides, smaller resonances are better visible.

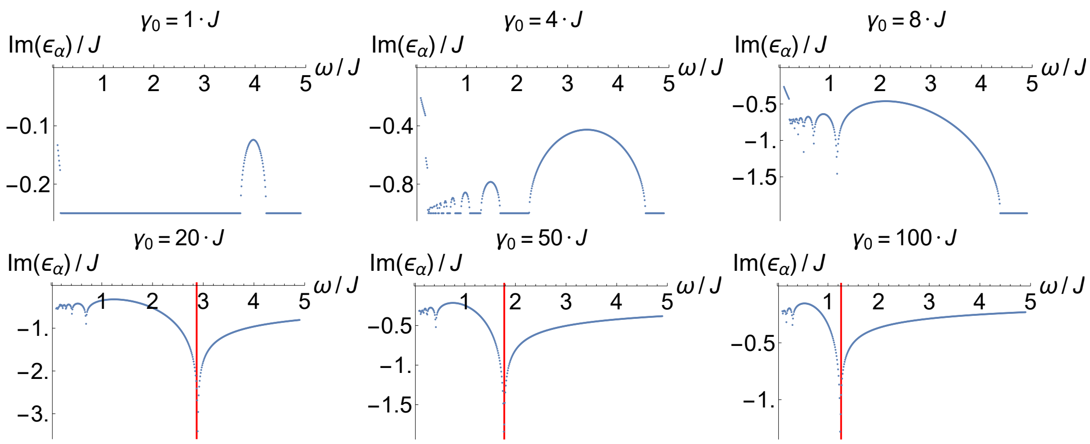

Figure 5.12: Imaginary part of the eigenvalue ε_α with the largest imaginary part depending on the frequency ω for different γ_0. The red lines in the second row correspond to the resonance frequency depicted in figure 5.8.

Unfortunately, this approach does not give us an analytic expression for the quasienergies depending on the parameters of our system. Therefore, it does not allow us to analyse the reason for the occurance of these resonances at specific frequencies in detail.

6 Summary

After a short introduction to closed and open quantum systems, we reviewed the most important properties of the Lindblad master equation. This equation was one method we used to describe the model of dissipatively driven waveguides. We also tried to characterize the problem with a non-hermitian Hamiltonian.

In general, our waveguide model includes the hopping of the photons to the nearest neighbour sites as well as a possible loss of photons on every waveguide. In our description, this dissipation is the result of the dissipator $\mathcal{D}(\rho)$ in the Lindblad master equation and, in the other formalism, of the non-hermitian part of the Hamiltonian.

First of all, we had a look at a homogeneous system. Thus, the hopping amplitude $J(t)$ and the dissipation strength $\gamma(t)$ are the same for every waveguide. Furthermore, we used periodic boundary conditions. With the help of a Fourier transformation of our bosonic operators we managed to solve the Lindblad master equation in the Heisenberg picture and to diagonalize the non-hermitian Hamiltonian. We saw that the total number of particles in the system decays exponentially with γ in the time-independent case. For a time-dependent γ, we observe an exponential decay with an associated time-dependent strength. Both formalisms lead to this result.

After that, we concentrated on a system, which consists of two waveguides but only one of them having the property of losing photons. In this case, we solved the Lindblad master equation for the whole density matrix restricted to one-particle states. Starting with a time-independent dissipation, one finds for small values of γ an exponential decay of the total number of particles, which is similar to the decay in the corresponding homogeneous system. But now the decay goes with half of the dissipation strength then before and contains some oscillations. Besides, the system gets into the Zeno limit for $\gamma \geq 8J$. Hence, the hopping of the particle to the dissipative site is suppressed. The decay is in this case proportional to $1/\gamma$. Note that this behaviour is a consequence of the chosen initial condition. If we consider that the particle starts on the dissipative site we would not observe the Zeno effect. Again, the results of both formalisms agree.

In the last step, we changed the dissipation for the same two waveguide system and made it periodic in time. In this case, we solved the Lindblad master equation in the Heisenberg picture numerically and compared it to the solution of the non-hermitian Hamiltonian using Floquet theory. While the behaviour of the system was again similar to the homogeneous one up to a factor of two for small values of γ, we observe for bigger ones resonances at specific frequencies ω. At these frequencies, the particle gets lost very quickly in comparison to other ones. These resonances also occur in the imaginary part of the eigenvalues of the Floquet modes. It is still unclear why these resonances occur at exactly these frequencies. But in general, the periodic change of the dissipation strength let the system switch between being in the Zeno limit and allowing the hopping of the particle to the dissipative site.

A Appendix

A.1 Derivation of our Lindblad equation in the Heisenberg picture

Plugging in the operators $b_i^\dagger(t)b_j(t)$ in the Lindblad equation in the Heisenebrg picture 2.2 we get

$$\frac{\mathrm{d}}{\mathrm{d}t}(b_i^\dagger(t)b_j(t)) = \underbrace{-i\sum_{l=1}^{L} J_l(t)[b_l^\dagger(t)b_{l+1}(t) + b_{l+1}^\dagger(t)b_l(t), b_i^\dagger(t)b_j(t)]}_{*}$$
$$+ \underbrace{\sum_{l=1}^{L}\gamma_l(t)\left(b_l^\dagger(t)b_i^\dagger(t)b_j(t)b_l(t) - \frac{1}{2}\{b_l^\dagger(t)b_l(t), b_i^\dagger(t)b_j(t)\}\right)}_{**}.$$

Using the commutation relations we then have

$$\begin{aligned}
* = -i\sum_{l=1}^{L} J_l(t)\Big(&b_l^\dagger(t)\left([b_{l+1}(t), b_i^\dagger(t)]b_j(t) + b_i^\dagger(t)[b_{l+1}(t), b_j(t)]\right)\\
&+ \left([b_l^\dagger(t), b_i^\dagger(t)]b_j(t) + b_i^\dagger(t)[b_l^\dagger(t), b_j(t)]\right)\cdot b_{l+1}(t)\\
&+ b_{l+1}^\dagger(t)\cdot\left([b_l(t), b_i^\dagger(t)]b_j(t) + b_i^\dagger(t)[b_l(t), b_j(t)]\right)\\
&+ \left([b_{l+1}^\dagger(t), b_i^\dagger(t)]b_j(t) + b_i^\dagger(t)[b_{l+1}^\dagger(t), b_j(t)]\right)\cdot b_l(t)\Big)\\
= -iJ_{i-1}(t)b_{i-1}^\dagger(t)b_j(t) &+ iJ_j(t)b_i^\dagger(t)b_{j+1}(t) - iJ_i(t)b_{i+1}^\dagger(t)b_j(t) + iJ_{i-1}(t)b_i^\dagger(t)b_{j-1}(t)
\end{aligned}$$

and

$$\begin{aligned}
** &= \sum_{l=1}^{L}\gamma_l(t)\Big(b_l^\dagger(t)([b_i^\dagger(t), b_l(t)] + b_l(t)b_i^\dagger(t))b_j(t) - \frac{1}{2}b_l^\dagger(t)b_l(t)b_i^\dagger(t)b_j(t) - \frac{1}{2}b_i^\dagger(t)([b_j(t), b_l^\dagger(t)]\\
&\qquad + b_l^\dagger(t)b_j(t))b_l(t)\Big)\\
&= \sum_{l=1}^{L}\gamma_l(t)\Big(-\delta_{i,l}b_l^\dagger(t)b_j(t) + b_l^\dagger(t)b_l(t)b_i^\dagger(t)b_j(t) - \frac{1}{2}b_l^\dagger(t)b_l(t)b_i^\dagger(t)b_j(t) - \frac{1}{2}\delta_{j,l}b_i^\dagger(t)b_l(t)\\
&\qquad - \frac{1}{2}b_i^\dagger(t)b_l^\dagger(t)b_j(t)b_l(t)\Big)\\
&= \sum_{l=1}^{L}\gamma_l(t)\Big(-\delta_{i,l}b_l^\dagger(t)b_j(t) + \frac{1}{2}b_l^\dagger(t)b_l(t)b_i^\dagger(t)b_j(t) - \frac{1}{2}\delta_{j,l}b_i^\dagger(t)b_l(t) - \frac{1}{2}b_l^\dagger(t)([b_i^\dagger(t), b_l(t)]\\
&\qquad + b_l(t)b_i^\dagger(t))b_j(t)\Big)\\
&= -\frac{1}{2}b_i^\dagger(t)b_j(t)(\gamma_i(t) + \gamma_j(t))
\end{aligned}$$

and so equation 4.1 follows.

A.2 Eigenbasis for diagonalization of Lindblad equation

The eigenvectors of the matrix M defined in 5.1.1 are given by:

$$\begin{pmatrix} 0\\0\\0\\0\\0\\0\\0\\0\\1 \end{pmatrix} \begin{pmatrix} -\frac{1}{2}\\ -\frac{i\gamma}{8J}\\0\\-\frac{1}{2}\\0\\0\\0\\0\\1 \end{pmatrix} \begin{pmatrix} 0\\1\\1\\0\\0\\0\\0\\0\\0 \end{pmatrix} \begin{pmatrix} 0\\0\\0\\0\\0\\0\\ \frac{-i\left(\gamma+\sqrt{\gamma^2-64J^2}\right)}{8J}\\1\\0 \end{pmatrix} \begin{pmatrix} 0\\0\\0\\0\\ \frac{i\left(\gamma+\sqrt{\gamma^2-64J^2}\right)}{8J}\\1\\0\\0\\0 \end{pmatrix} \begin{pmatrix} 0\\0\\0\\0\\0\\0\\ \frac{i\left(-\gamma+\sqrt{\gamma^2-64J^2}\right)}{8J}\\1\\0 \end{pmatrix}$$

$$\begin{pmatrix} -\frac{\gamma+\sqrt{\gamma^2-64J^2}}{2\gamma}\\ \frac{-4i\left(\gamma^2J-64J^3+\gamma J\sqrt{\gamma^2-64J^2}\right)}{\gamma\sqrt{\gamma^2-64J^2}\left(\gamma+\sqrt{\gamma^2-64J^2}\right)}\\ \frac{4i\left(\gamma^2J-64J^3+\gamma J\sqrt{\gamma^2-64J^2}\right)}{\gamma\sqrt{\gamma^2-64J^2}\left(\gamma+\sqrt{\gamma^2-64J^2}\right)}\\ \frac{-32J^2}{\gamma\left(\gamma+\sqrt{\gamma^2-64J^2}\right)}\\0\\0\\0\\0\\1 \end{pmatrix} \begin{pmatrix} 0\\0\\0\\0\\ \frac{-i\left(-\gamma+\sqrt{\gamma^2-64J^2}\right)}{8J}\\1\\0\\0\\0 \end{pmatrix} \begin{pmatrix} -\frac{\gamma-\sqrt{\gamma^2-64J^2}}{2\gamma}\\ \frac{4i\left(-\gamma^2J+64J^3+\gamma J\sqrt{\gamma^2-64J^2}\right)}{\gamma\sqrt{\gamma^2-64J^2}\left(-\gamma+\sqrt{\gamma^2-64J^2}\right)}\\ \frac{-4i\left(-\gamma^2J+64J^3+\gamma J\sqrt{\gamma^2-64J^2}\right)}{\gamma\sqrt{\gamma^2-64J^2}\left(-\gamma+\sqrt{\gamma^2-64J^2}\right)}\\ \frac{32J^2}{\gamma\left(-\gamma+\sqrt{\gamma^2-64J^2}\right)}\\0\\0\\0\\0\\1 \end{pmatrix}$$

Note that the order corresponds to the one of the eigenvalues in 5.1.1.

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